cycles

—A MONTHLY REPORT

October 1951

FOUNDATION FOR THE STUDY OF CYCLES
New York, New York

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cycles

—a monthly report

The Official Bulletin of the Foundation for the Study of Cycles

VOLUME II NUMBER 8 OCTOBER 1951

The Director's Letter

Dear Member:

At the present time we know little more about cycles than was known about chemistry in the days of Boyle, Cavendish, Priestly, and Lavoisier, the fathers of modern chemistry, who made their pioneering discoveries about 150 years ago. We know no more about cycles than was known about•germs in the days of Leeuwenhoek, who discovered bacteria.

What do we know about cycles anyway?

What we do not know

Let us start by saying what we do not know.

First, we do not yet know the exact wave length of any of the multitude of cycles observable in natural and social science, except of course the cycle of the day, the cycle of the year, the cycle of the tides, and other astronomical cycles.

Second, we do not yet know all the cycle lengths which exist, nor do we know the laws, if any, governing the interrelationship between cycle lengths. That is, in spite of hints, we do not yet know if there are " families " of cycles, and if the discovery of one cycle implies the existence of other cycles a half as long, a third as long, a fourth as long, etc., or related in some other fixed manner.

Third, we do not yet know the ultimate cause of any of the cycles except the astronomical cycles.

Fourth, our mathematics are not yet adequate to tell us the statistical significance of the cycles we discover. That is, we cannot yet know the number of times out of a thousand, let us say, that a cycle of given regularity could occur as the result of " chance "—that is of random and unrelated forces just happening to come at reasonably regular time intervals.

Fifth, as yet we cannot forecast on the basis of cycles except in terms of probability. That is, we cannot say that such and such will or will not happen. We can merely say that if the observed cycles continue the odds are such and such in favor of a certain behavior.

What we do know

Now, what do we know?

First of all, we know that the fluctuations of hundreds and hundreds of phenomena in natural science and in human affairs fluctuate with a beat or rhythm which is too regular to be easily the result of random forces.

Second, many cycles have kept on coming true after discovery—in some instances cycles have kept on coming true for more than eighty years—thus proving beyond any reasonable doubt that the observed rhythmic behavior has significance.

Third, although some of the observed rhythms may be the result of sporadic causes which just happen to come at more or less equal time intervals, it is inconceivable that all the observed rhythms are purely accidental. And, if any of the rhythms are real (i.e., not the result of random forces), we have stumbled upon facts of the utmost scientific importance.

Fourth, the wave lengths of the cycles discovered in natural and social phenomena seem in many instances the same, thus suggesting a natural environmental cause for at least certain of the cycles observed in human behavior.

Fifth, the wave lengths of the cycles discovered on earth often correspond to the cycles discovered in the sun, thus suggesting an astronomical cause for at least some of the terrestrial cycles.

Sixth, most phenomena evidencing cyclic behavior act as if they were being acted upon by a number of different cyclic forces all operating at the same time.

Application

How can we apply our knowledge of cycles?

In the present limited state of our knowledge I feel that we should apply what we know, first, for general background information, and second, on an actuarial basis, for actual profit.

I think you understand well enough what I mean by " general background " without further elaboration, but the term " on an actuarial basis " may not be clear.

As you know, an actuary is an expert who calculates insurance risks and premiums. The basic principle of insurance underwriting consists of spreading the risk. The insurance people know they will lose on some of their commitments and they take these losses in their stride because they know that on the average they will come out all right. The trick of success is first to know your ratios and second to operate with a large enough sample so that the law of averages has a chance to govern.

Similarly with a knowledge of cycles, instead of trying to figure out what the market is going to do, I think that in the present state of our knowledge the best way to put our knowledge of cycles to practical use is to find some reliable short term cycles which can be depended upon to yield consistently satisfactory results on the average, when played on a round trip basis.

To oversimplify it, suppose for example you find a short term cycle of, let us say one month in length which, when traded by means of a long position for two weeks and a short position for two weeks shows a consistent record over a very long period of time of 8 gains to 4 losses, with gains amounting to 3 times losses, and with 20% per annum net profit after losses.

If you were to operate on such a cycle, you would count on small losses as a matter of course, but, if you had isolated a true cyclic factor, or factors, you could count on more profits than losses, regardless of whether the trend of market was up or down. That is, you could spread your losses as the insurance company does and average them with enough gains to come out satisfactorily. If the market were going up your long gains would be greater, if it were going down, your short gains would be greater, but on the round trip it would make no difference one way or the other.

With the knowledge of several short term cycles your profits could be expected to increase.

* * * *

As I look over this issue it seems pretty dull. Who cares about 37-year cycles? Most of us will be dead 37-years from now. And anyhow, the article deals with sunspots, which seem mighty remote from every day living. And who wants to know about moving averages?

What we want to know is whether business next year is going to be good or bad, when the next war is going to start, whether we are going to have a severe winter, and whether stocks are going to go up or down tomorrow—or next week anyway.

I want to know these things as much as you do, and I will try to make next month's report of greater immediate usefulness. But in justification of the present report I want to remind you that in cycle study as in other things we have to creep before we can walk. If we try to walk too soon we may fall down and hurt our faces!

Very cordially yours,

Edward R. Dewey

Director

Research

THE 37-YEAR CYCLE

1. In the variation in the length of time between sunspot maxima (since 301 A. D.).
2. In sunspots with alternate cycles reversed (since 1749).
3. In the frequency of the aurora borealis (since 385 A. D.).
4. In the frequency of Chinese earthquakes (since 200 A. D.).
5. In the floods of the river Nile (since 622).
6. In the frequency of severe winters in Europe (since 330 A. D.).
7. In temperature at New Haven (since 1781).
8. In the growth of Arizona pines (since 1460).
9. In the abundance of lynx (since 1735).
10. In wheat prices (since 1265).
11. In cotton prices (since 1731-32).
12. In common stock prices.
    (a) In railroad stock prices (since 1831).
    (b) In industrial stock prices (since 1871).

Summary

The seeming existence of a 37-year cycle in 12 different phenomena suggests behavior of the most fundamental importance. It would be almost impossible to find a cycle of the same length in solar phenomena, aurora, weather, earthquakes, floods, tree rings, and prices—in some instances for more than 1000 years—as a result of random forces.

ONE of the more important cycles with which we have to deal seems to be about 37 years in length. If the work of various investigators proves to be correct, this cycle is present in a dozen or more different phenomena from sunspots to common stock prices, and over periods of time which in some instances extend back into history for 1000 to 1500 years.

I find this cycle particularly interesting not only because it is so important but because it has been found both on earth and on the sun, thus indicating the possibility of some astronomical cause. As yet no one knows what this cause might be.

Cause not planetary

The cause does not seem to be connected with the planets.

The observed length of this cycle does not correspond to the length of time required by any of the planets to go around the sun (see **Cycles—a Monthly Report**, April 1951, page 135) nor does it correspond to any simple fraction of any of these lengths.

Neither does the length of the cycle correspond to the length of time required by any two of the planets to " get in step" with each other (**Cycles—a Monthly Report**, April 1951, page 137), nor any simple fraction of any of these lengths.

Other Hypotheses

Of course, you must not rule out the possibility that the 37-year cycle in solar phenomena may be of slightly different wave length from the 37-year cycle on earth, and hence unrelated to the earthly cycles, or that the 37-year cycles on earth and on the sun have the same length merely by accident. (When cycles in different phenomena have the same wave length this fact merely suggests an interrelationship; it does not of itself prove it.)

Moreover, it is always possible that the long sunspot cycles which come on the average at 37 1/2 year intervals may not be the result of a 37 1/2-year force at all, but may be the result of other cyclic forces—perhaps some harmonic of 37 1/2 years— in combination with the 11-year cycle.

However, assuming for the moment that the length of the force is 37 1/2 years, that the length of this cycle in the sun proves to be exactly the same as the length of the corresponding cycles on earth, and that there is an interrelationship, the behavior could perhaps be caused by waves of energy coming from space which influence both the sun and ourselves, or it could perhaps be caused by variations of solar radiation of some sort which have their repercussions here on earth, or it could be some combination of the two.

What all this boils down to is that as yet we do not know anything about cause; at the present time we can merely guess.

But whatever the cause, a cycle of about 37 years seems to be with us, for better or worse, apparently on the sun and in at least ten different phenomena here on the earth. We must know more about it if we are to understand the various phenomena in which it exists, and if we are to predict the future behavior of such phenomena.

The twelve phenomena

Incidentally, in case you did not read the subtitle of this article, the twelve phenomena in which this behavior has been observed are as follows:

(a) In the variation in the length of time between sunspot maxima since 301 A. D.
(b) In sunspots with alternate cycles reversed since 1749.
(c) In the frequency of the aurora borealis since 385 A. D.
(d) In the frequency of Chinese earthquakes since 200 A. D.
(e) In the floods of the river Nile since 622 A. D.
(f) In the frequency of severe winters in Europe since 330 A. D.
(g) In temperature at New Haven since 1781.
(h) In the growth of Arizona pines since 1460.
(i) In the abundance of lynx since 1735.
(j) In wheat prices since 1265.
(k) In cotton prices since 1731-32.
(l) In common stock prices.
    (1) In railroad stock prices since 1831.
    (2) In industrial stock prices since 1871.

If I were to tell you about the 37-year cycle in all these phenomena in one article it would take up an entire issue of our monthly report. I have therefore divided the material into twelve chapters and will print it as a continued story, putting as many chapters as possible into each of various future issues until you have the entire account.

1. THE 37-YEAR CYCLE IN THE VARIATION IN THE LENGTH OF TIME BETWEEN SUNSPOT MAXIMA, 301 A. D. — 1928 A. D.

Summary

A cycle which on the average is 37-1/2 years long but which varies in length has been discovered in fragmentary sunspot data from 300 A. D. to the present time.

For our knowledge of the 37-year cycle in the variations in the length of time between sunspot maxima we are indebted to Mr. H. W. Clough (pronounced Cluff). In fact, much of our knowledge of the 37-year cycle comes from Clough's work.

Mr. Clough was for many years meteorologist of the U. S. Weather Bureau. He is now retired and living at Castile, New York, near Rochester. I am sure he would be glad to talk cycles with you if you are passing near his home and find yourself able to stop by to see him.

Let me explain what we mean by the expression " variations in the length of time between sunspot maxima. " Then let me explain what Mr. Clough found in these variations.

Sunspot Maxima

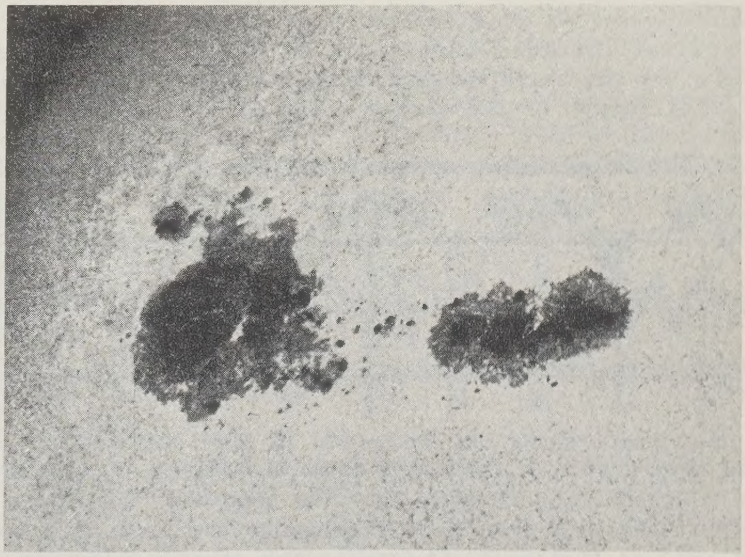
Sometimes the sun is almost completely free from spots. A few spots then appear, then more and more spots until, in the course of several years, there may be dozens and dozens of spots. Then the number of spots begins to diminish until eventually, in about 11 years, the surface of the sun becomes almost completely free from spots again. These spots seem to be vortexes of cooler gas, but as yet no one knows just why they occur, and particularly why they occur in cycles.

Although the length of the sunspot cycle on the average is about 11 years, the length of any one cycle may be anywhere from 9 to 13 years, and in extreme instances sunspot cycles have been known to be as short as 7 or as long as 16 years from trough to trough or crest to crest.

Is there any regularity in the coming of these longer cycles? The work of Mr. Clough indicates that there is, and that, on the average long sunspot cycles come about 37.5 years apart.

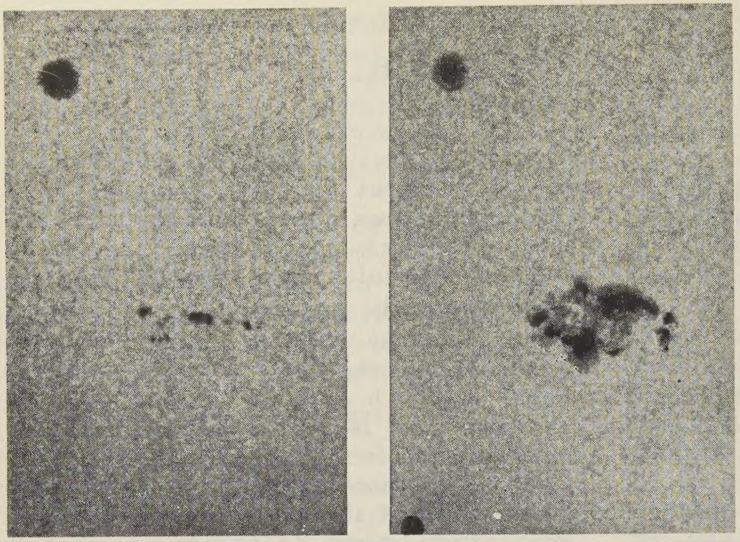
Early Sunspot Records

The number of spots on the sun have been counted, month by month, with increasing accuracy from 1749 to date. However, before that date there are fragmentary data running back to the earliest observed spot of which we have any record, the one of 165 B. C. From these fragmentary records it has been possible to



The great spot group of February, 1946. This is the largest spot group recorded since Galileo first turned a telescope upon the sun in 1611. Photographed at the 60-foot tower telescope of the Mount Wilson Observatory on Feb. 2, 1946.

The entire earth could be droppen into any one of the vast craters shown. It would be completely swallowed up and never would be missed.



Twenty-four hour development of a spot group. Disc in lower left hand corner of right hand picture represents the comparative size of the earth. Aug. 18 and 19, 1917. (Mount Wilson Observatory.) Think of craters bigger than the earth developing in a single day!

reconstruct dates of probable maxima, back at least as far as 301 A. D.

For further particulars you may want to refer to the article "Sunspots before 1750" on pages 253 and 254 in our September 1951 report, and to Clough's own paper on the subject, "The 11-year sun-spot period, secular periods of solar activity, and synchronous variations in terrestrial phenomena," as published in the **Monthly Weather Review**, Vol. 61, April 1933, pages 99-108.

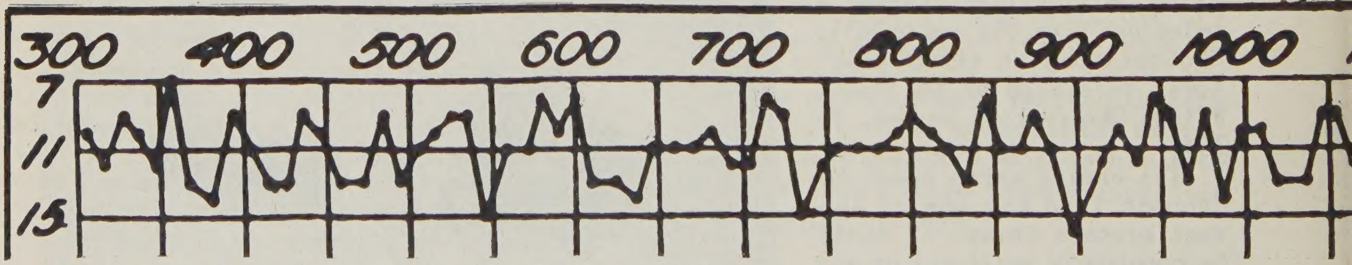
Having the dates of estimated maxima it is easy to subtract each from the one following to get the length of the cycle, measured from crest to crest. Thus, as the first maximum comes in year 301 and the second comes in year 311, the length of the first recorded sunspot cycle is 10 years. As the third maximum comes in year 323 the length of the second cycle is 12 years, and so on.

A 37-1/2-Year Cycle

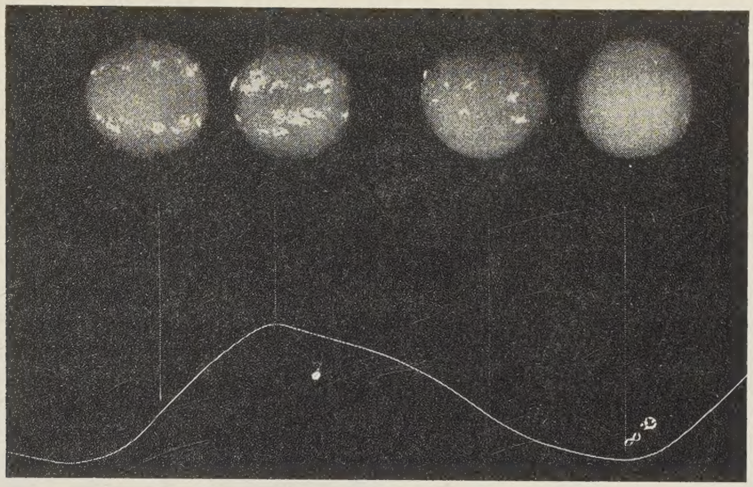
Mr. Clough plotted these various intervals in a chart reproduced herewith as Fig. 1. When the lengths of the various cycles have been plotted this way it is easy to see that there is a tendency for long cycles and short cycles to alternate in a cycle which turns out to average 37.4 years from end to end or 37.5 years when its length is determined by means of least squares.

Qualifications

In evaluating the significance of Clough's observations you should remember that he did not have available to him actual dates of sunspot maxima, but merely estimates based on fragmentary observation and upon early auroral data (which can be used because auroras are known to be associated with sunspots). But errors of the sort which may be in the data would tend



This chart shows the variation in the length of the sunspot cycle, measured from maximum to
mum, from 300 A. D. to date. That is, there is a point on the curve for each cycle and these
are plotted in accordance with the length of the cycle represented. The scale is shown at the
The horizontal center line represents the typical or average 11-year cycle. Note that the
reversed so that points representing short cycles are shown above the center line and



The curve represents the relative number of sunspots in a typical sunspot cycle as it unfolds over a period of about 11 years. Starting with practically no spots the number of spots increases for several years to a maximum, and then decreases again.

The four views of the sun taken at the times indicated by the vertical white lines show the progress of the spottedness over the course of a cycle. The pictures are calcium spectroheliograms. The bright areas show the changing position and size of the flocculi, which accompany the spots. In these pictures you cannot see the actual spots; they are underneath the flocculi.

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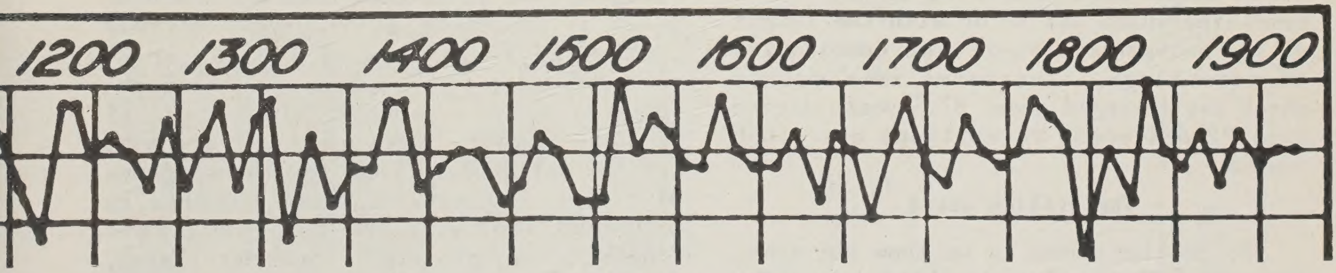
to counterbalance each other, and, unless the data are much less reliable than Clough believed them to be, would have little or no bearing upon his conclusions.

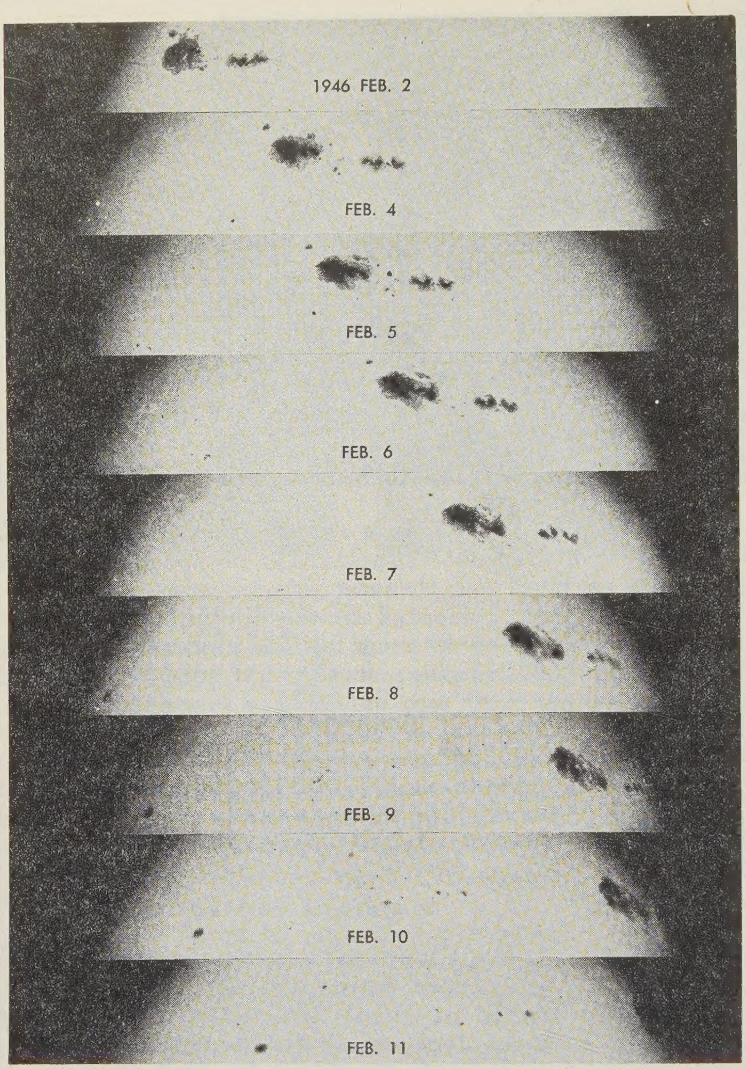
Bear in mind also that I am not reporting to you upon the results of my own observations, but upon the work of another investigator. I hope sometime to have the time to check Clough's findings by the use of methods which had not been developed when his work was done, and to come to my own conclusions in respect to this cycle in these figures. When I have done this work I shall make my own report to you in regard to this cycle, and give you my own opinion as to its exact length, either in an article in our monthly report or in the **Journal of Cycle Research.**

A cycle of varying length

Mr. Clough also found that this 37-1/2-year cycle seemed to vary its length in a cycle of about 300 years. That is, for about four cycles the length of the 37-1/2 year cycle tends to be less than 37-1/2 years in length, then for about four cycles the length tends to be more than 37-1/2-years in length and so on. But that observation perhaps belongs in a story



senting long cycles are shown below the center line.
e points are connected by lines to bring out the fact that these cycle lengths in turn fluctuate cycle which averages 37½ years over the period of the study (1650 years). The ends of each cycle ircled. Note that this is not a uniform cycle of 37½ years, but that the cycle is first longer hen shorter than 37½ years in a cycle which on its part is about 300 years in length.



Varying aspects of the great sunspot group of February, 1946, as it was carried across the disc by the solar rotation. This is the largest spot group ever recorded. It had an area of 6.3 billion square miles, or more than 30 times the area of the earth's surface.

These giant spot groups were easily visible to the unaided eye through smoked glass or thick haze at sunset, and in some localities attracted wide attention. Photographed at the 60-foot tower telescope of the Mount Wilson Observatory.

This picture and the three preceeding pictures and much of the material in the captions are reproduced from **Astronomy** by William T. Skilling and Robert S. Richardson, copyright 1947 by Henry Holt and Company of New York. **Astronomy** is an excellent book of nearly 700 pages and over 300 pictures. If you are interested in the subject I suggest that you may want to add this book to your library.

about the 300-year cycle and not in an account of the 37 1/2-year cycle itself.

What does all this mean?

It means that **something** on the sun—something which has to do with the length of the sunspot cycle—or some combination of somethings, fluctuates in a period which has averaged about 37.5 years during some 42 different repetitions over 1650 years.

The problem ahead

The problem ahead is to know the exact length of the cycle and whether it is a simple cycle or a compound of two cycles, one slightly over 37.5 years and one slightly under 37.5 years. Unless we know its exact length we cannot compare it intelligently with the so called 37.5-year cycle on earth.

Also we need to know what this something is that fluctuates this way and what makes it fluctuate in 37.5-year periods instead of in periods of 30, 40, or 50 years. We also need to know by what mechanism it affects terrestrial affairs, if it does. However, these questions are more for the astronomers than for the students of cycles. Our work as cycle students is finished when we give the exact wave length, shape, amplitude, calendar timing, and probable significance of the cycle. In this particular instance all this work has yet to be done, in spite of the work of Mr. Clough reported upon above which has blazed the way.

STOCK MARKET CYCLES BY "PREDIX"

Summary

A comprehensive, scholarly, cycle analysis of stock market prices has just been published by Vedder Hughey under the trade name of " Predix. " Hughey has isolated, synthesized, and projected 19 cycles in stock market prices from 1854 to date. The techniques developed by Hughey are ingenious and significant. His work is worthy of serious consideration.

YOU will join with me, I am sure, in being much interested and impressed by an analysis of stock market cycles recently published by Mr. Vedder Hughey of Coral Gables, Florida, under the trade name of " Predix. "

Mr. Hughey's publication consists of a large (32"x 22") three color chart together with a 24-page booklet.

What the chart shows

The chart shows quarterly average common stock prices from 1854 through the second quarter of 1951; it plots the trend and diagrams 19 different cycles found by Mr. Hughey in these quarterly averages; and it shows the amplitude and timing of each cycle. The chart projects the trend and each of the discovered cycles through 1972. The chart also shows a combination or synthesis of the trend and all of these 19 cycles. It compares this synthesis with actual stock market behavior 1854-1951 and projects the synthesis through 1972 to show the combined effect if the indicated cycles are real and continue.

The booklet

The booklet tells something of the methods used, gives a test forecast 1928-1952 based on data no later than 1915, and gives tables showing the actual numerical value of the index (in logs), the combination of cycles and trend, and the residue. It also gives dates and numerical values of the turning points of each of the 19 waves.

The job impresses me as honest, scholarly, competent, and worthy of the most careful consideration by all serious students of cycles.

The cycles found

In spite of a gracious release of copyright in our favor printed on his chart, it would not be fair to the enormous amount of work put into this project by Mr. Hughey to tell you what his projection shows for the future or the amplitudes and dates of his various cycles; but I am sure he would not object to my telling you the wave lengths of the cycles he discovered. I am going to do so below in comparison with my own preliminary work of 1943-44 reported upon for you in the March, June, and September 1951 reports.

Indicated Wave Length

Dewey Preliminary Reconnaissance	Predix
2.58 years	
	2.82 years
2.96 "	
3.40 "	3.38 "
3.79 "	
4.89 "	4.88 "
5.50 "	
6.07 "	6.00 "
6.86 "	
	7.22 "
	7.70 "
8.17 "	
	8.50 "
9.20 "	9.20 "
	10. "
	10.65 "
11.00 "	
12.00 "	12.00 "
	13.50 "
14.50 "	
	16. "
	18.3 "
	19.5 "
21.00 "	
	23.25 "
	30.00 "
	37.50 "
	51.50 "

Several things are of interest in this comparison.

The 19 1/2-year cycle

First of all, note that Hughey's fifth longest wave is 19.5 years long. This wave was not discovered by me in my preliminary work of 1943-44 but I subsequently discovered it in railroad stock prices and reported upon it to you in our September report on page 255. You will remember that I set the length at 19.4 years, but I am sure Mr. Hughey and I will not quarrel over the half of one per cent difference. Hughey's work was published before my work was published and his work was done before my work was done, so clearly, he should get credit on all counts for the discovery of this cycle. On the other hand, my work was completed before Hughey's work was published or I knew any of his findings and thus provides an absolutely independent check.

The 37-1/2-year cycle

Similarly, in regard to the 37-1/2-year cycle. My work on this cycle in these figures is not yet finished, but it has gone far enough so that it is very clear to me that there has been a cycle of this general order of magnitude in stock market prices. I shall report to you upon my own work on the cycle in connection with the continued story about the 37 1/2-year cycle in twelve different phenomena which starts with this issue. But it is interesting that Hughey should come up independently with this exact length. Thus in two instances my current and more detailed work confirms Hughey's work. This fact tends to substantiate the reliability of both his work and mine.

Other cycles found

In five other instances our work also agrees. We both get 3.40-year cycles (Hughey 3.38), 6.07-year cycles (Hughey 6.00), 9.20-year cycles, and 12.00-year cycles.

In addition to these seven instances, there are nine cycles indicated as possibilities by my reconnaissance which are not verified by Hughey's work and twelve cycles indicated by Hughey's work not included in my work so far.

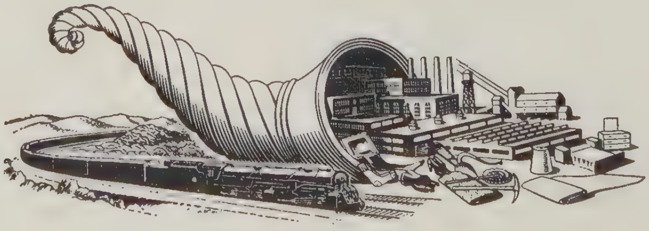
First contact

My first contact with Mr. Hughey—a member of the Foundation—came on May 5, 1949, when he wrote in answer to my first quarterly letter to lay members, to say that he was making a section moving average analysis of stock market prices by means of I. B. M. punch cards, that he had several columns on his cards open, and that he would be glad to turn them over to me for scientific work in the event I was interested.

Mr. Hughey engulfed in work

I sent him the promised material, but although I wrote him in 1950 and again in 1951 to ask how he was coming on, I heard nothing further from him until July 20 of this year when an enormous package of I. B. M. tabulation sheets arrived in the mail, followed quickly by a letter and by complimentary copies of the Predix chart and its accompanying booklet.

I am much impressed by the quantity and quality of Mr. Hughey's work. Where his work and mine agree we uphold each others hands, as it were. Where we differ we encourage each other, and other investigators, to conduct further research. And by the very fact of differing we put everyone on notice that the present state of our knowledge of cycles is only partial and incomplete, albeit growing with great rapidity.



MARECHAL'S STOCK MARKET FORECAST OF 1933

IN his book **New Methods for Profit in the Stock Market**, Mr. Garfield A. Drew reproduces on page 162 a copy of a forecast of stock market prices as calculated and drawn in late 1933 by George Marechal, formerly of Montreal, Canada. Mr. Drew states that one of the original copies of this forecast has been in his personal possession since 1935.

In my copy of Drew's book, published in 1948, Marechal's forecast is included only through 1948, but through the kindness of Mr. Drew, the remaining values of Marechal's original forecast have been made available to you and are included in the chart shown on the opposite page.

As each year in Marechal's forecast was divided into six parts, Mr. Drew added the actual fluctuations of the Dow-Jones Industrial Averages on the same basis by taking the high and low for two month periods.

Remarkable Similarity

As Mr. Drew says, " Clearly, the pattern of the forecast and the actual pattern of the market miss many times in detail and exact timing. Nevertheless, the broad picture of the trends from 1934 through 1947, at least, is remarkably similar. The basic downtrend from 1936-1937 to 1942 is plain, and likewise the uptrend from 1942 to 1946, although the latter shows up as a much more zigzag pattern in the forecast than was actually the case. Thus, the year 1944 by itself, for example, appears as a down period, whereas it was really an up year.

" When the year 1947 ended, the Dow-Jones Industrial Average had spent sixteen months within a 16% price range—one of the narrowest and longest such movements on record. It is interesting, therefore, that the forecast also showed sixteen months of similar back and forth movement within—except for one quick dip—exactly the same price range.

" As far as the situation at the time the comparison in Figure 10 (June, 1948) ends is concerned, it is evident that if the broad accuracy of the preceding 14 years is to be maintained, 1948 must—on the whole—witness a rising price level. A definite downtrend going substantially into new low territory by the year-end would produce a greater discrepancy between the forecast pattern and the actual course of prices than at any other time in the record. The fact remains to be seen at this writing, but in line with his original forecast made years before, Marechal always insisted that 1946-47 was not a 'bear market,' but an interruption in a long upward trend, comparable to the break and market hesitancy during 1926 in the long upswing from 1921 to 1929. "

In a letter to me dated July 12, 1951 Mr. Drew said, speaking of the six additional points on the curve which he provided, " I should say, however, that this is nothing Marechal has recently added, but was on the original. Of course, his time element was too far ahead and the actual period of hesitation between 1947 and 1949 was longer than he originally showed. However, at least the basic conception of a new high by a wide margin was correct.

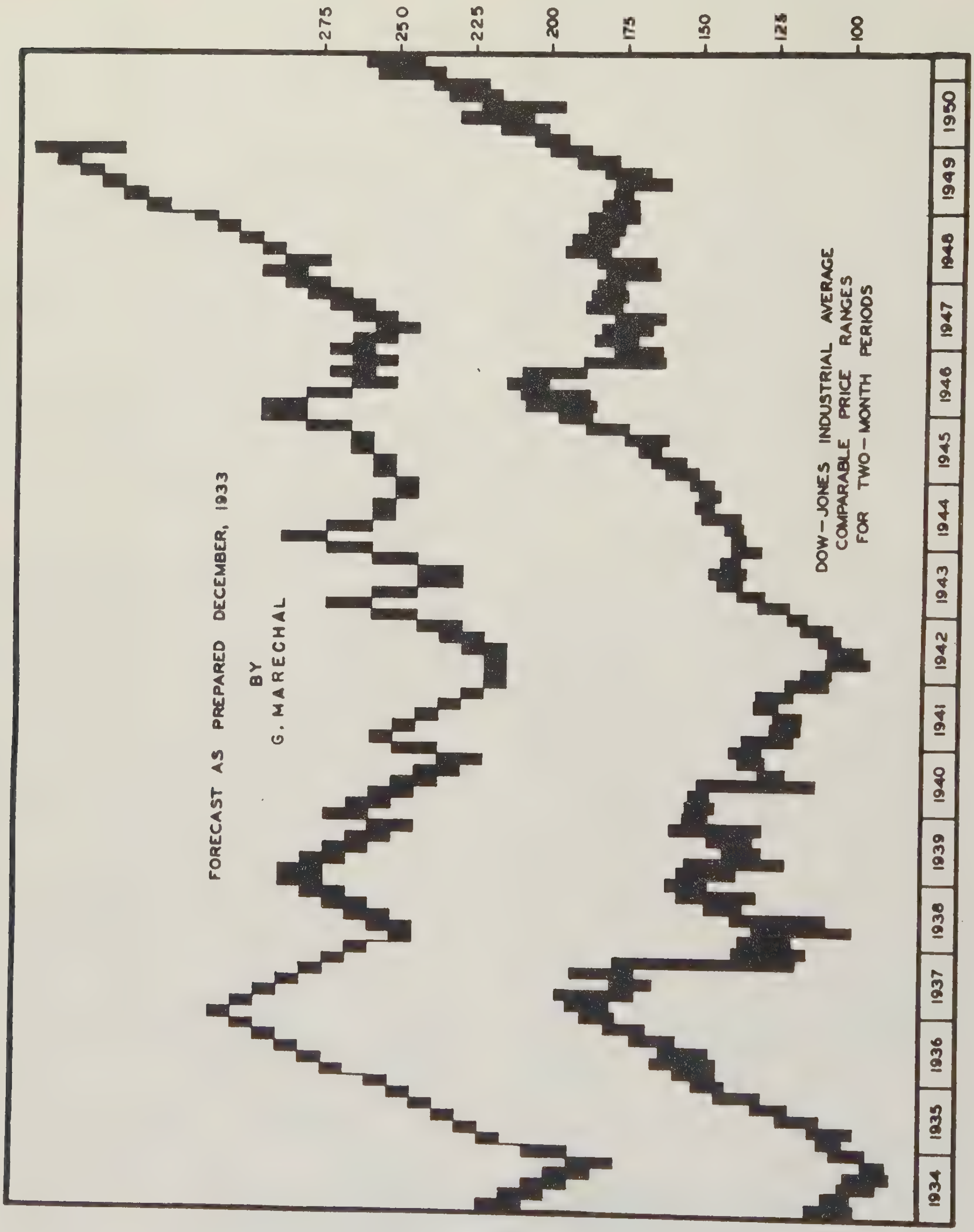
" The old original forecast made some years ago does not go beyond what is shown on the enclosed copy. However, I know that on what Marechal worked out several years ago, a severe decline follows the recent peak—apparently to what would be 150 in the Industrial Average. "

On Aug. 2, 1951 Drew wrote me " Incidentally, although Marechal is long-term bearish, he thinks there will probably be a new high first. "

Cycles Included

This projection is included here first to indicate that, if you know enough, economic fluctuations can be at least partially predicted and second because, in making this forecast, Marechal took into account " all important cycles " as well as various trend and resistance lines projected into the future.

The chart, up to and including 1948, and the quotations concerning it are reproduced from Mr. Drew's book through the courtesy of the Matcalf Press, Boston, Massachusetts, which holds the copyright. Incidentally, if you are interested in the stock market and do not own this book, I think you would be well advised to get a copy. You can get it from any book store or through the publication office of the Foundation. The price is $4.50.


FORECAST AS PREPARED DECEMBER, 1933
BY
G. MARECHAL
DOW-JONES INDUSTRIAL AVERAGE
COMPARABLE PRICE RANGES
FOR TWO-MONTH PERIODS
275
250
225
200
175
150
125
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1934
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1936
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1938
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1940
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1948
1949
1950

TEN IMPORTANT QUESTIONS

Dear Sir:

I received your letter of welcome to the Foundation, for which I thank you.

I am intensely interested in the study of cycles although I have done no work along that particular line. I would like to, however. . .

(1) If cycles in the stock market are caused by extremes of optimism and pessimism of people, do they coincide with cycles of inanimate data like tree rings or animals? Or is there an underlying principle for both?

(2) If cycles are " the science of prediction " of what practical use are they with the margin of error in time that you now find? Since the error in time varies with the length of the various cycles, is it possible after observing a turn in a short cycle to compare the situation with a longer cycle and arrive at reliable conclusions as to how far the longer cycle has progressed in comparative amplitude? It is hard to make my point clear on this one.

(3) In the market cycle, I watch the action progress by working down from the main trend to the minor trend. That is, if the main trend is up, the intermediate trend up, and the minor trend turns down, is the intermediate trend also turning or will there be one or more minor up-trends?

(4) It would seem to me that all the smoothing, etc. you do with the data increase the error in accuracy. It may put out the fluctuations but it seems to get away from the main consideration which is to know in advance if possible when a cycle is going to change?

(5) If you know as you do that the present method of constructing cycles seldom results in coinciding with the actual tops and bottoms, and that the actual cycles vary in length of time, why not recognize that fact and construct the projected cycles of irregular length by means of combining them?

(6) Do cycles of various lengths show or coincide with the actual tops and bottoms of the data in accordance with amplitude? That is, if a 41 month cycle in the stock market is due to make a top say three years from now but the 9 year cycle top is not due for say six years, does that indicate the probabilities are that the top six years from now will be the more important one?

(7) I have found that the characteristics of short trends are similar to those of the longer trends? Does that same principle apply to cycles? If so, and if you lack sufficient data for study, why not construct daily or even hourly cycles of the market for analysis. By the way, have you any indications of the length of cycles that have been determined daily or hourly? I would like to work on this subject if you think it would be constructive. I have plenty of back figures to use. There seems to be indications of 21-41 and 81 days and also hours.

Should market or calendar days be used in this work?

(8) Moving averages of the market show a varying rate of change as they approach tops and bottoms. Why cannot this factor be used to determine the particular phase or relative position a cycle is in?

(9) Paragraph 5, page 202 of June 1951 Report states " A third factor that makes the actual low come ahead or after ideal timing ". Does not that prove that the secret of more accurate results comes from combining two or more cycles into one cycle of irregular timing?

(10) Paragraph 2, column 2, same page, states " The effect of the 54 year rhythm

etc." Is it a mathematical proven fact that its effect in the last year of the 27 is no greater than during any other one-year period? If so, can it be proved mathematically that its effect is greater than the 9 year cycle during that same period?

Well, Dr. Dewey, I guess I have already taken too much advantage of your kindness, so I will close this letter. I enjoy reading your reports very much indeed.

Thanking you for any comments you may have.

Sincerely,
Providence, R. I. William B. Freeman

THE ANSWER

Dear Mr. Freeman:

It is a pleasure to try to answer your questions, in order, to the best of my ability.

Answer to Question Number One

1. I used to think that cycles in the stock market were caused by extremes of optimism and pessimism of the people who bought and sold stocks. Something in the atmosphere, let us say, made a group of people bearish or bullish all at the same time. And this indeed may be the truth for some of the cyclic fluctuations in the market.

On the other hand, as you can see by reference to the six periodograms prepared by Mr. E. S. C. Coppock and published in our March '51 report, different stocks have very different cycles, and of course we know different industries have very different cycles too.

Market Cycles May be Reflection of Industry Cycles

I am therefore inclined to believe that at least some of the cycles we observe in the market are the reflection of cycles in the industries involved rather than mass movements of optimism and pessimism of traders as a whole

Do these cycles in the market coincide with the cycles that are found in natural science?

Market Cycles and Natural Science Cycles Often Have Same Length

We do not yet know enough to answer this question with assurance. For the past 50 years or so the most important medium-length cycle in the market has been about 41 months in length. Professor Huntington finds a cycle of this length in atmospheric electricity. Sterne of Harvard found cycles of this length in solar radiation. I am informed that a cycle of this length is an important element in certain relatively successful long range weather forecasts (but I do not know this of my own knowledge) and Garcia-Mata finds a cycle of this length as a minor constituent of sunspot numbers.

For the past 50 years this cycle has also been an important characteristic of the number of international battles.

Lemmings are also supposed to migrate in a cycle of about 3 1/2 years but I have never been able to assure myself that these migrations fall in to a 41 month pattern.

As far as I know no one has ever tried to see whether or not there is a 41 month cycle in the alternate thickness and thinness of tree rings.

Or take another example:

There is very definitely a 6 year cycle present in the stock market as well as in the production and/or sales of many large American corporations. I devoted almost all of our December 1950 report to this cycle and showed it to be present in each of some 25 different companies. This 6 year cycle is also present in tree rings at Fairlee, Vermont, in barometric pressure at New York and in sunspots with alternate cycles reversed.

I could go on at great length and fill several pages to give illustrations of cycles which are present in the market and for which there seem to be exact counterparts in nature.

On the other hand I could give you the length of many cycles in the market for which, as yet, no natural science counterparts have been found. And I could give you many cycles indisputably present in nature which have not yet been found in the market, for example, the 9.6-year cycle which has dominated the abundance of many forms of wild life in Canada since at least 1730.

Answer to Question Number Two

2. You asked about the practical use of the knowledge of cycles. If one attempts to rely on the knowledge of any one cycle there are, as you say, difficulties. The actual crest is almost certain to come either before or after ideal timing, but it seems to me that even so there is a certain very definite advantage in knowing what the probabilities are. That is, it seems to me important to know that in the majority of instances the highs tend to come so many months or weeks or days one way or the other of perfect timing and to have similar information for the lows. It seems to me valuable to know that at the exact time of high such-and-such a value can be expected in a majority of instances.

Actuarial Basis of Operation Recommended

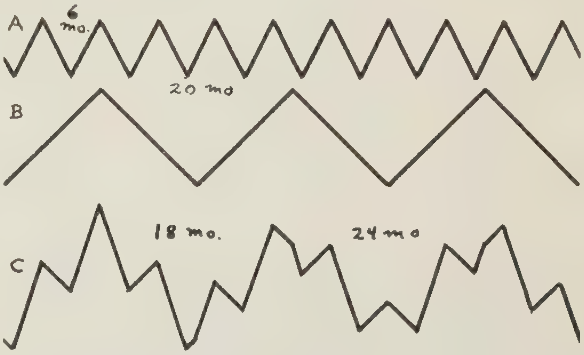
Your question is akin to the question, "What is the value of life expectancy tables?" The answer is, no value at all, if you want to know the exact date on which your millionaire Uncle Henry is likely to die. However information of this sort is very useful indeed if you are running an insurance company and want to play the averages.

The corollary of all this is, as a practical matter, if one insists upon limiting oneself to a knowledge of one cycle, that one should choose a very short-term cycle where one can operate on an actuarial basis and come out satisfactorily on the average regardless of how one may come out on any individual transaction.

Use of Several Cycles is Preferable

However, why should one limit oneself to a single cycle? By taking a number of cycles into account, one can get a much closer approximation to the truth and can expect to know in any given instance whether, by reason of the short-term cycle, a crest (or trough) is likely to come ahead of or after ideal timing.

To answer the second part of this question, it is not only possible but in my opinion essential to adjust for the short-term cycles, to see how the long-term cycles are progressing, and to adjust for the long-term cycles to learn the progress of the trend.



If a series of figures acts as if it were influenced by a cyclic force A, and also by a cyclic force B, we can get a more accurate representation if we combine the cycles, as in C. Also, if the cycles are real and continue, C if projected will give a much better forecast.

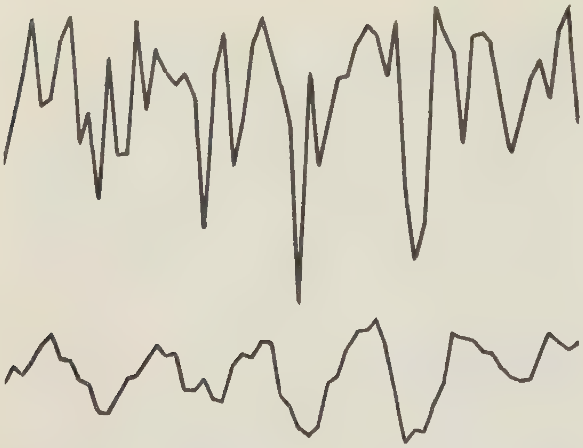
Answer to Question Number Three

3. You asked, "If the main trend is up and the intermediate trend up and the minor trend turns down, is the intermediate trend also turning?"

If you have your short cycles correctly determined, you can adjust for them just as you would for a seasonal pattern and discover whether or not the intermediate trend is turning or is still continuing.

Answer to Question Number Four

4. Smoothing of data is an expedient to minimize some of the fluctuations so that the other fluctuations can be seen more clearly. A short-term smoothing enables you to see the cycles of medium length more clearly. A medium length smoothing enables you to see the long cycles more clearly. Having determined the length of the various cycles and having combined and projected them, one should then compare behavior with the actual unsmoothed data.



A, unsmoothed curve. The cycle is hard to see.

B, smoothed curve. A cycle becomes readily visible.

Answer to Question Number Five

5. Your question illustrates what in my mind is the correct method of proceeding. It is the method I used in the projection printed on pages 220 and 221 of the June 1950 issue.

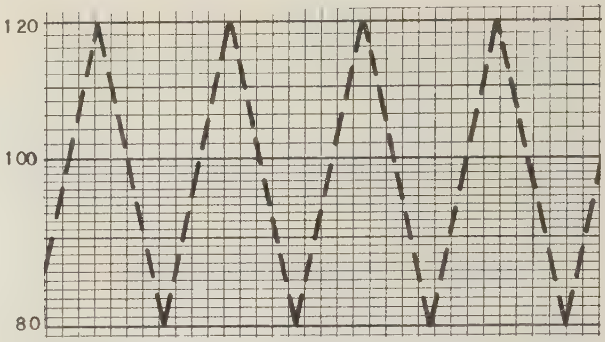
By using 10 different cycles all of different lengths and amplitudes, I made a projection in 1944 which, considering the crude preliminary nature of the work done at that time, seems to me to have turned out quite satisfactorily over the course of the 7 years that have elapsed since it was made.

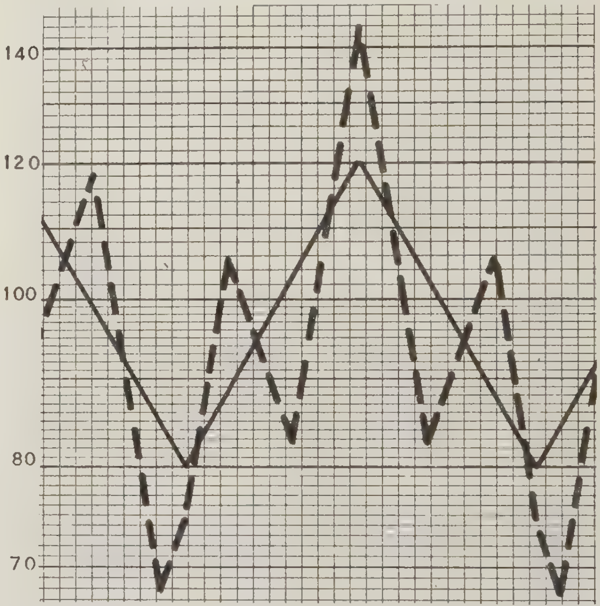
Answer to Question Number Six

6. As nearly as I can determine it, the amplitudes of the various waves of a cycle remains constant in their percentage effect. If the amplitude of the 9-year cycle at crest is 120% of trend and the amplitude of the 41 month cycle at crest is also 120% of trend and if the crests come together, the combined amplitude would be 120% of 120% or 144% of trend.

To get the combined effect of two cycles one has to determine the value of each cycle for each month as a percentage of trend and multiply these values together.

In the hypothetical example you give the top 6 years from now would be definitely more important than the top 3 years from now.





This chart shows the combination of a hypothetical 9-year or 108-month wave and a hypothetical 41-month wave. The combined effect is shown by a broken line.

Answer to Question Number Seven

7. For the most part the cycles I have studied seem to have the same shape. They seem to go up and down in straight lines when the percentage variation from trend is plotted on log paper. For the most part they are symmetrical with the lows exactly midway between the highs.

On the other hand, I have not yet been able to discover any laws governing the interrelationship of cycle length. Some people feel that cycles exist in what is sometimes called triple progression. That is, if you have one cycle of 2 years in length you will have another cycle of 6 years in length, another of 18 years in length, and so on.

Other people believe that cycle lengths bear harmonic relationship to each other. That is, if you have a cycle 60 months long, you will have other cycles half that length or 30 months; a third of that length or 20 months; a fourth of that length or 15 months and so on.

I myself have found a number of instances to substantiate each of these two theories and have also noticed, with what seems to me to be too great a frequency to be mere chance, a tendency for monthly cycles to have their counterparts in annual cycles (or visa versa). That is, you find what seems to be 9.18-month cycles and 9.18-year cycles; 5.55-month cycles and 5.55-year cycles. However, I feel our knowledge of cycle length is still too incomplete and too crude to justify generalizations.

Too Soon for Theories

The truth is that there are so many cycles and that the lengths have been measured so crudely that it is possible to spin almost any kind of a theory that you want and find a good deal of backing for it.

You will remember that Tyndall once said that an idea in a empty head had the time of its life because it could rotate in any orbit it choose without having any facts to bump into.

I have done very little work on daily and hourly stock market cycles but I should think work in this direction might be very fruitful.

My own belief is that the price cycles are the results of environmental forces. Therefore, in the absence of evidence to the contrary, I should think that calendar days instead of market days should always be used. To get a homogeneous series of figures, values for Saturdays, Sundays, and holidays can be interpolated.

Answer to Question Number Eight

8. You speak of the varying rate of change of moving averages as they approach tops and bottoms. Is this behavior due to a nonlinear shape of the cycle or is it due to the fact that the actual top or low is usually brought about by a combination of cycles?

However, regardless of which of these circumstances may be the truth, I should think this factor could very definitely be used to advantage for orientation. I know at least one man who operates a very successful forecasting business largely on the use of this principle.

Answer to Question Number Nine

9. The entire sentence that you started to quote reads, " A third factor that makes the actual low of a major price cycle come somewhat ahead of or after the ideal timing is the fact that there are a number of other shorter cycles in prices riding on the back of the big cycle. "

As I implied, the secret of more accurate results lies in combining several cycles into one pattern of irregular timing.

Answer to Question Number Ten

10. The entire paragraph to which you refer reads as follows, " The effect of the 54-year rhythm in prices over a 27-year span is very important indeed, either on the down side or on the up side, but its effect in the last year of the 27 is no greater than its effect during any other one-year period starting with 1925. "

This statement is based on my observations that most cycles seem to average out to a linear or zigzag shape. On the other hand, Mr. W. C. Yeatman of Los Angeles states accurately that if cycles are truly zigzag in shape, moving percentages of these cycles should have flat tops. This is an obvious truism. Mr. Yeatman then asks me if I have ever seen moving percentages of cycles evidencing flat tops and I have to answer that I have not. This fact casts doubts on the accuracy of my observations in regard to cycle shape.

As for the second part of your question, it is very easy to determine approximately the over-all amplitude of each cycle and, by dividing this over-all effect by the number of months or years in half a cycle, to get the effect for any single month or year and to see the relative importance per unit of time.

I trust this letter answers your questions and that if you have more questions you will write to me again.

Very cordially yours
Edward R. Dewey
Director

Questions and Answers

DEFINITIONS ASKED FOR

Q. I am mixed up about the precise meaning of the words **cycle, rhythm, wave,** and **periodicity.**

A. These terms are very confusing because we all use them so loosely.

Strictly speaking a **cycle** is a coming around again to the place of beginning. It comes from a Greek word meaning circle. Thus the life cycle of the moth runs from egg to larva to pupa to moth and back to egg again. The business cycle runs from one depression through a boom back to a depression again.

Rhythm comes from a Greek word meaning measured time. It is applied to phenomena which recur with a beat. Thus music and poetry and drum beats and heart beats have rhythm. So do cycles the highs or lows of which recur with reasonable regularity.

An absolutely regular cycle is called a **periodicity.**

One cycle, from crest to crest or trough to trough is called a **wave.**

The trouble comes from the fact that in actual practice these strict meanings are not often adhered to.

For example, one speaks of the 9.2-year cycle in the stock market or the 9.2-year wave in the stock market when one really should say " the 9.2-year rhythm " (if one is speaking about the tendency of the market to fluctuate with a beat of an average interval of 9.2-year between crests (or troughs)); or else should say " the 9.2-year periodicity " (if one is speaking about the idealized behavior which is exactly 9.2-years from crest to crest or trough to trough).

Our concern is with Regular Cycles

When you and I speak about cycles we almost always mean regular or rhythmic cycles—rhythms. In fact that is the only kind of cycles in which we are interested. However many people are interested merely in the sequence of events during the course of a cycle—say from low to low. Such people may have no interest at all in trying to see if there is any regularity of timing between lows, yet they are quite correct—such is the looseness of our language—in describing their interest as the study of cycles, too.

Hundreds of books have been written about the business "cycle ", using the work quite correctly, in which there is not one sentence to imply that the cycles repeat with a tendency toward regularity.

WHAT JUSTIFICATION FOR CLAMING A 9.2-YEAR CYCLE IN THE STOCK MARKET ?

Q. Granted that there is some correspondence between the smoothed stock market figures shown in Curve B on pages 206 and 207 of your June report and your 9.2-year zigzag, (a) what justifications do you have for saying that there is a 9.2-year cycle? (b) Couldn't the observed behavior have come about by accident—that is as the result of random forces? (c) In fact couldn't you find " cycles " even in numbers taken at random from a telephone book, let us say?

A. Let me answer your questions in reverse order.

Cycles in Random Numbers

(c) Very definitely yes, you can find rhythmic cycles or tendency toward regularity in random numbers, such as numbers from a telephone book.

The whole matter is one of degree,—the extent towhich the various waves conform to a perfectly regular pattern and the numbers of times that the pattern has continued.

You can easily fit a perfectly regular cycle (periodicity) to random numbers, but the correspondence between the random numbers and the regular cycle is not great, especially if the series of random numbers is long, and such cycles therefore have no significance.

Cycle in the Market Perhaps Random

(b) Of course the behavior shown in the stock market chart could have come about as a result of random forces.

Something could have given the market a stimulus in 1860. Another something could have come along in 1870. A third something (entirely unrelated) could have come along in 1878, and so on. It could be that we have here a series of ten different somethings which just happened, at random, to come at or near 9-year intervals. The proper question is not " Could this be random fluctuation ", but rather, " What is the **probability** of randomness? How many times out of a hundred could one get cycles or waves as regular as these purely as a result of a random distribution of forces? "

The 9.2-year Cycle a Fact

(a) Your first question asks my justifications for saying that there is a 9.2-year cycle in these figures.

I think we may be using the word cycle in a slightly different way. I think that you feel that by the use of the word "cycle " I mean to imply that the waves are interrelated, are the result of a common cause, and therefore will continue, but I do not.

Would it help things if I changed the wording to say that there have been oscillations in these figures (which is a pure statement of fact), and that a perfectly regular 9.2-year pattern more or less fits these oscillations (which is also a simple matter of fact)? That is all I mean when I say that there is a 9.2-year cycle in these figures.

It follows that my justification for saying that there is a 9.2-year cycle is (has been) present in these figures is the simple fact that it is true—if we both use the words involved in the same way to mean the same thing.

Real Question is its Significance

The question properly therefore is not whether there has been a 9.2-year cycle in common stock prices for the past 95 years, but whether the degree to which the observed behavior conforms to an arbitrary rigid 9.2-year pattern is enough to justify the opinion that the behavior is the result of some underlying rhythmic force which, having continued for 95 years (throughout ten cycles) may be expected to continue into the future.

I get cross when people question the existence of cycles, because that is nonsense. But I never get cross when they question the **significance** of the cycles (oscillations-waves), because such scepticism is very wholesome and searching and we need more of it.

OTHER QUESTIONS

Question: Has any work been done on the weighting of economic cycles, particularly in relation to the 41-month, the 9-year, the 18 1/3-year, and the 54-year cycles?

Answer: All these four cycles have been weighted with great accuracy in many different economic time series.

You can get the approximate average strength in a number of instances by referring to the scales at the left of various charts in **Cycles—The Science of Prediction**.

Question: Why are the trends in the "basic" industries of significance to all of our people and not merely the stockholders, workers, and executives of these industries?

Answer: Because the basic industries themselves are such large consumers, and because their stockholders, workers, and executives also loom large in the entire economy as consumers and as sources of capacity.

Additions to the Library

"The 11-Year Sunspot Period, Secular Periods of Solar Activity, and Synchronous Variations in Terrestrial Phenomena," by H. W. Clough, reprinted from the **Monthly Weather Review**, Vol. 61, April, 1933; 9 pages, 7 charts, 3 tables, & references. Gift of Mr. Clough.

It is impossible to do justice to Mr. Clough's paper in a few notes. Consideration of the material presented is a **must** for all students of long term cycles.

In addition to the 11-year period Clough discusses the 37-year period, the 83-year period, the 300-year period and the 1400-year period.

The 37-year period is reported upon as found in the increase and decrease of length of the sunspot cycle, in the frequency of auroras, in the frequency of severe winters, in the frequency of Chinese earthquakes, in wheat prices in England expressed as a percent of the secular (long term) trend, in the annual growth of Arizona pines, and in the floods of the River Nile. See pages 277 to 282 of this issue for the first installment of an article on this cycle.

The 83-year period is reported upon as found in the variation in the length of sunspot cycles, in the frequency of auroras, in the frequency of severe winters, in the frequency of Chinese earthquakes, in the growth of sequois per decade, and in the low stages of the River Nile.

The 300-year period is reported upon as found in the length of the 37-year period, in the length of the 83-year period, in the average of 11-year residuals, in the raininess of Europe, Belgium, and China, in the severity of winters of Europe, China, and Belgium, in the frequency of ice in Danish waters, in the low stages of the Nile, in the growth of sequoias, in the abundance of auroras, in the frequency of earthquakes in China and in the whole earth, in the lag of the maxima of 37-year cycles of severe winters after the dates of the maximum acceleration of the 11-year solar cycles, and in the lag of the maxima of the 83-year cycles of severe winters.

The 1400-year period is reported upon as found in the variations in the length of the 11-, 37-, 83-, and 300-year solar periods, in rainfall in Europe and Asia, in temperature in Europe, and in the thickness and thinness of tree-rings.

• • •

Quantitative Methods in Politics, by Stuart A. Rice, Alfred A. Knopf, New York, 1928, 331 pages, 51 tables, 22 charts. Gift of Dr. Rice.

This book, concerned as it is with quantitative measures of political activity, provides many tables of figures of value to the student of cycles.

In addition, in the last three chapters Dr. Rice discusses cycles in party "turnover" in the New Jersey assembly vote, cycles in the reelection of experienced congressional incumbents, and cycles in the amplitude of the electorial swing, i.e. landslides, one way or the other.

In this connection Dr. Rice calls attention to "the comparative regularity of the cycles of landslide indexes. The period of these seems to vary from eight or ten to twelve or fourteen years. The regularity disclosed could scarcely result from chance."

• • •

Cyclical Analysis—Industrial Common Stock Index, 1854-1951, together with 32"× 22" chart in three colors, by PREDIX (Vedder Hughey), 1951, Coral Gables, Florida; 24 pages, 20 tables, 2 charts, plus the supplemental chart referred to above. Gift of Mr. Hughey.

See review of this material on page 283 of this issue.

CYCLE ANALYSIS: THE MOVING AVERAGE

By
Edward R. Dewey

Technical Bulletin No. 4

FOUNDATION FOR THE STUDY OF CYCLES

1950

FOREWORD

Many of the scientists and others who wish to study the cycles in their respective fields are not statisticians. Nor do they wish to wade through text books of statistics to find out how to deal with the cycles which they find in the data with which they currently work. Moreover, even if they were willing to spend the necessary effort, it would not profit them too much, for the text books simply do not explain enough about the various techniques of cycle analysis to provide adequate help; the elementary text books often ignore the subject almost entirely.

It therefore seems desirable for the Foundation to issue a series of simple informal Technical Bulletins on the subject of cycle analysis. It is hoped that these bulletins will place adequate working knowledge of the different statistical techniques within easy reach of the many workers who have come across cyclic behavior, and who wish to study the cycles they have found.

These bulletins are intended primarily for natural scientists such as biologists, geologists, physicians, dendrochronologists, climatologists, astrophysicists, agronomists, hydrologists; for social scientists such as sociologists, economists, anthropologists; for business statisticians not previously too familiar with analyses of this sort, and for classroom use in courses of statistics.

Bulletin No. 1 outlined the subject. Bulletin No. 2 gave one test for the significance of a discovered cycle. Bulletin No. 3 described the Hoskins Time Chart. This bulletin deals with the moving average.

Future bulletins will deal with the moving percentage, the moving section average, autocorrelation, simple and multiple harmonic analysis, the analysis of non-harmonic functions, the periodic table, the periodogram, bracket analysis, optical, mechanical, electrical, and punch card methods of resolution, and various other tools that should be in the kit of all workers in this field. Some of the titles sound formidable, but the subject matter is really very simple, and will be spelled out so that anyone with a knowledge of simple arithmetic will be able to perform all the necessary operations, except those involving the use of machines.

The following pages should be thought of as a preliminary draft being circulated for comment, criticism, and suggestions. The reader is requested to contribute all he can—by way of questions, criticism, and notations of any errors that may have crept in — toward making the next edition plainer and more easily comprehensible.

E. R. D.

January, 1950

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SUMMARY 322

Cycle Analysis: The Moving Average

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The moving average is a mathematical tool of great use to students of cycles. As there is confusion in the minds of some people in regard to the use of this tool, it seems wise to issue a bulletin on the subject.

Some sections of this bulletin are merely a restatement of what you can find in any good text book of statistics. In other sections, however, you will find material, some of which is not, as far as I know, available readily, if at all.

I. DEFINITIONS AND DESCRIPTION OF METHODS

(The Simple Arithmetic Moving Average)

Averages

Everybody knows that an **average** is a typical value which tends to sum up or describe a number of figures. There are at least five different kinds of averages commonly used by statisticians; but the one which ordinary folk think about when they hear the word **average** is the one computed by adding all the items together and dividing the total by the number of items. Thus, if we have four items, 10, 12, 11, and 13, the average of these items is 10 + 12 + 11 + 13 (46) divided by 4, or 11½. (Statisticians call an average computed this way the **arithmetic mean**, but you do not need to remember this term, because I shall not use it again.)

Time Series

An arrangement of numbers is called a **series**. When the numbers with which we deal represent events which occur one after another in time, the arrangement is called a **time series**. Thus, in the example above, if 10, 12, 11, and 13 represent the price of cotton for each of four consecutive years, or represent the number of accidents on each of four consecutive days, you would call the numbers by this name—a time series.

You could still average the numbers and say, for example, that the average price for all four years was 11½ cents, or that during the period there was an average of 11½ accidents per day, as the case might be.

You could also say that the average price for the first three years was 11 cents, (10 + 12 + 11 (33) divided by 3) and that the average price for the last three years was 12 cents (12 + 11 + 13 (36) divided by 3)..

Moving Averages

A moving average is merely a succession of averages secured from a series of numbers by dropping the first number (item) in each group averaged and including the next number in the series after the group, thus obtaining the next group to be averaged, and so on.

Thus, when you averaged the first three numbers of our time series (10, 12, and 11) and got 11, and then dropped the first number (10) and added the fourth number (13) and averaged again and got 12, you were constructing a moving average. Easy, wasn't it?

Because you were averaging three items at a time, you would call the result a 3-item or 3-term moving average. If the items represented yearly values you would call the result a 3-year moving average. If the items represented daily values, you would call the result a 3-day moving average.

Moving Totals

The moving total is the series of successive totals from which the moving average is computed.

For example: When, above, you added 10, 12, and 11 to get 33, and then added 12, 11, and 13 to get 36 (as a step in the task of getting 11 and 12, the two terms of the moving average), you were computing a moving total.

It was so easy that you did it without knowing it!

The moving total, like the moving average, should be posted in a table or plotted on a chart against the middle item of the group of items being totalled, as will be explained below.

Plotting or Posting Moving Averages

Each item or term of a moving average is always properly posted or plotted against the center of the group of items being averaged.

Many otherwise intelligent people fool themselves into thinking that if they plot or post an average against the last figure of the group of figures being averaged they somehow are getting later values. Of course this is nonsense.

If the values for 1933, 1934 and 1935 were 10, 12, and 11 respectively, the average for these three years is 11, whether we say 11 for the three years **beginning** in 1933, or 11 for the three years **centering** on 1934, or 11 for the three years **ending** in 1935. In **talking** about an average, we could choose any one of the three ways with equal propriety, as long as we made it clear which way we had chosen. But when averages are posted to a table, or plotted as a point on a chart, they **must** be posted or plotted against the middle of the group of figures being averaged, otherwise convention will be violated and, much more important, distortions are introduced into all further work. (The reasons for this will appear later.) Let me repeat, moving averages must **always** be posted or plotted against the central item of the items being averaged.

Two Examples

To make the process doubly clear, let us work out teo examples:

TABLE 1.
COMPUTATION OF 3-YEAR AND OF 7-YEAR MOVING AVERAGES

		COMPUTATION OF A 3-YEAR MOV. AVER.		COMPUTATION OF A 7-YEAR MOV. AVER.	
	A	B	C	D	E
YEAR	DATA	3-YEAR MOVING TOT. OF COL. A	3-YEAR MOVING AV. OF COL. A (COL. B÷3 OR COL. B x 1/3)	7-YEAR MOVING TOT. OF COL. A	7-YEAR MOVING AV. OF COL. A (COL. D÷7 OR COL. D x 1/7)
1933	10	-	-	-	-
1934	12	33	11	-	-
1935	11	36	12	-	-
1936	13	40	13 1/3	91	13
1937	16	43	14 1/3	98	14
1938	14	45	15	109	15 4/7
1939	15	46	15 1/3	127	18 1/7
1940	17	55	18 1/3	149	21 2/7
1941	23	69	23	173	24 5/7
1942	29	87	29	-	-
1943	35	104	34 2/3	-	-
1944	40	-	-	-	-

It is obvious that with a 3-year moving average there are no values to place against the first and the last items of the series. With a 7-year moving average there are no values to place against the three first and the three last items of the series. In constructing a moving average one always loses one or more terms at each end.

Formulae

The formula for a 3-year moving average is

$$MA_b = \frac{a + b + c}{3}$$

where a to c represent successively each three consecutive terms of the data and MA_b stands for the 3-year moving average to be posted against the central term, b.

The formula for a 7-year moving average would be

$$MA_d = \frac{a + b + c + d + e + f + g}{7}$$

where a to g represent successively each seven consecutive terms of the data and MA_d stands for the 7-year moving average to be posted against the central term, d.

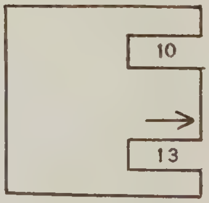
Mechanical Details of Computation

As has been explained, to get a 3-year moving average, one first computes a 3-year moving total, and divides each item of the moving total by 3.

To get the first figure of the moving total, add together the first three items of the data. To get the next figure of the moving total you subtract the first item of the data, and add the fourth. This process gives you the sum of items 2, 3, and 4. You proceed in this way successively.

In actual practice it is hard to pick out which items to add and which to subtract. You get mixed up.

To make the calculation foolproof, cut two slots out of a card or piece of paper so as to expose the first and fourth items in the series, but not the second and third. In our example in Table 1, it would look like this: (The lines **between** slots must always be one less than the number of terms in the moving average.)



Place this screen over the data so that the first item (in this instance 10) appears in the upper slot and the fourth item (in this instance 13) in the lower one.

Now, from 33, the sum of the first three figures, already posted in Col. B, subtract whatever appears in the upper slot (10) and add what you see in the lower slot. This gives you 36 which you enter in Col. B opposite the arrow. (The arrow is placed against the middle figure of the three items whose total is obtained by this method.)

```
 10 *
 12
 11
 33 S
 10 -
 13
 36 S
 12 -
 16
 40 S
 11 -
 14
 43 S
 13 -
 15
 45 S
 16 -
 17
 46 S
 14 -
 23
 55 S
 15 -
 29
 69 S
 17 -
 35
 87 S
 23 -
 40
104 *
=====
 29
 35
 40
104 *
=====
```

Now, slip your screen down a line so that 12 shows in the upper slot and 16 in the lower one.

From 36, subtract 12 and add 16 to get 40, the third item of your moving total.

Continue in this way until you have dropped 23 and added 40 to come up with the final item in the moving total, namely 104.

Now, add together the last three items of the data-- 29 + 35 + 40, to get 104 as a check on the accuracy of your work.

If you use an adding machine with direct subtraction, your tape will look like the figures shown to the left:

I find it better to run the entire tape before posting any values to Col. B. One reason is that it is quicker to do your posting all at once. Another reason is, if you should make an error you will not need to erase from Col. B all the figures from the error forward.

In doing long columns of figures, I also find it a good idea to check every 50 or 100 items by adding up the proper number of items of the original data to see if the total agrees with the subtotal on my tape.

Now that we have our moving totals (Col. B) we compute the moving average either by dividing each figure of the moving total by 3 or by multiplying it by the reciprocal of 3. This latter method is often easier. The reciprocal of a number is 1 divided by the number. In this case it is .333333.

Actually, .333333 x 33, the first figure in Col. B is 10.999989 which, of course, rounds to 11. To get the 11 in the machine directly, I always record the last digit of the reciprocal as one more than it really is. In this case we would therefore have .333334 x 33 or 11.000022. The error is twice as large, but the excess is dropped anyway and this method saves the need of rounding.

Alternate (Short-Cut) Method

You can compute a moving average directly without computing the moving total. When a calculating machine is available, this method is usually preferable. The method is a little hard to describe but very easy to compute. Proceed as follows:

Place the reciprocal of the number of items of the moving average in the machine. As we are computing a 3-item moving average, we put in the machine the reciprocal of 3 which is 1/3 or .333333 (only, as above, we call it .333334). We lock this figure into the machine for the whole operation.

We first multiply this reciprocal by the first item of our data, 10, and obtain 3.33334. **Without removing this product**, we then multiply the reciprocal by 12 (add it in 12 times) and obtain a total of 7.333348. **Without removing the product** we then multiply the reciprocal by 11 to obtain a grand total of 11.000022 or 11, which is the first figure of our moving average. (1/3 of the first item + 1/3 of the second item + 1/3 of the third item is the same as the sum of the first three items divided by 3.)

We then remove 1/3 of the first figure by subtracting the locked-in reciprocal 10 times to get 7.666682 and multiply (add the reciprocal in) 13 times to obtain 12.000024 or 12, the second figure of our moving average. This process is continued right down the column until 23 times the reciprocal has been removed and 40 times the reciprocal has been added in to obtain 34.666736 or 34 2/3 for the moving average value for 1943. This value is checked by adding together 29 times the reciprocal, 35 times the reciprocal, and 40 times the reciprocal, or adding 29, 35, and 40 and dividing by three.

If we were computing a 7-year moving average, we would, of course, use the reciprocal of 7, which is .142858 (the last figure has been raised by one).. To get the first item (or term as it is more usually called) of our moving average, we add together the sum of this reciprocal times each of the first seven items of the data, and then add and subtract products of the reciprocal as above.

Moving Averages With an Even Number of Items

You may have noticed that so far we have talked exclusively about moving averages with an odd number of terms—3 or 7.

When we compute moving averages with an even number of terms such as 2 or 4, we run into a slight complication, due to the fact that the moving average must always be posted or plotted against the middle of the group of data being averaged, and the middle of an even number of items fall between two of the items.

We could post or plot a 4-year moving average **between** the years and this is sometimes done, as in the table on the following page:

Table 2.
Computation of a 4-Year Moving Average

	A	B	C
Year	Data	4-Year Mov. Tot. of Col. A	4-Year Mov. Aver. Of Col. A (Col. B÷4 or Col. B x .25)
1933	10		
1934	12		
		46	11½ (At position 1934½)
1935	11		
		52	13 (At position 1935½)
1936	13		
1937	16		

However, the results of this method of posting are very awkward to describe in words and preclude any comparison between the moving average and the original data. Therefore, in practice it is almost **universal** to compute a 2-item moving average of the even-term moving average in order to center the moving average exactly, thus:

Table 3.
Computation of a 2-Year Moving Average of a 4-Year Moving Average

	A	B	C	D	E
Year	Data	4-Year Moving Tot. of Col. A	4-Year Moving Av. of Col. A (Col. B÷4 or Col. B x .25)	2-Year Moving Tot. of Col. C	2-Year Moving Av. of Col. D (Col. D÷2 or Col. D x .5)
1933	10				
1934	12				
		46	11½		
1935	11			24½	12¼
		52	13		
1936	13			26½	13¼
		54	13½		
1937	16				
1938	14				

You note that the first item of the 2-year moving average of the 4-year moving average is centered exactly against 1935; the second item is centered exactly against 1936.

In practice one would have computed a 4-year moving total, then a 2-year moving total of the 4-year moving total, and divided these values by 8, thus:

Table 4.
Computation of 2-Year Moving Average of 4-Year Moving Average—Preferred Method

	A	B	C	D
Year	Data	4-Year Moving Tot. of Col. A	2-Year Moving Tot. of Col. B	2-Year Mov. Aver. of 4-Year Mov. Av. of Col. A (Col. C ÷ 8)
1933	10			
1934	12			
		46		
1935	11		98	12¼
		52		
1936	13		106	13¼
		54		
1937	16			
1938	14			

Or one would have used the short-cut method and computed a 4-year moving average directly, as explained above, without bothering with the 4-year moving total. One could also have computed the 2-year moving average of the 4-year moving average directly by the same means:

Table 5.
Computation of 2-Year Moving Average of 4-Year Moving Average—Short-Cut Method

	A	B	C
Year	Data	4-Year Mov. Average of Col. A, Computed Directly	2-Year Mov. Aver. of Col. B, i.e. a 4-Year Mov. Aver. of Col. A, Centered
1933	10		
1934	12		
		11½	
1935	11		12¼
		13	
1936	13		13¼
		13½	
1937	16		
1938	14		

Also, in actual practice, to save space, one posts the 4-year moving average in either the second or third position but marks it clearly to indicate that it is not truly centered as it should be, thus:

TABLE 6.
COMPUTATION OF THE 2-YEAR MOVING AVERAGE OF A 4-YEAR MOV.AVERAGE--POSTED AS IT IS DONE IN ACTUAL PRACTICE

	A	B	C
YEAR	DATA	4-YEAR MOV.AV. POSTED TO THE SECOND POSITION (CENTERED MINUS ½ YEAR)	2-YEAR MOV. AVER. OF 4-YEAR MO. AV. I.E. A CENTERED 4-YEAR MOV. AVER. OF COL.A
1933	10		
1934	12	46	
1935	11	52	12¼
1936	13	54	13¼
1937	16		
1938	14		

In any event, the final column is called a 2-year moving average of a 4-year moving average or, more usually, a centered 4-year moving average.

Formulae

For those who like to have relationships expressed in formula form, it may be stated that the formula for a centered 4-year moving average is:

$$MA_c = \frac{a + 2b + 2c + 2d + e}{8}$$

or more simply:

$$MA_c = \frac{\frac{1}{2}a + b + c + d + \frac{1}{2}e}{4}$$

where a to e represent successively each five consecutive items of the data and MA_c stands for the 4-year moving average to be plotted against c, the center term or item.

II. THE USE OF THE MOVING AVERAGE

This second section of the report will tell you how to use the moving average in statistical procedure, with particular emphasis upon its use in cycle analysis.

The moving average is used (a) to smooth time series, (b) to approximate the trend of time series, and (c), in cycle analysis, to help us (i) to separate cycles and (ii) to obtain a more exact estimate of the characteristics of each of the various cycles that may be present.

A. The Use of the Moving Average to Smooth Time Series

The chief use of moving averages in ordinary statistical procedure is for the smoothing of time series. As this use of the moving average as such does not particularly concern the cycle analyst, it will be touched upon here only very briefly.

A smooth curve is one which does not change its slope in a sudden or erratic manner. The student interested in smoothing time series is referred to Frederick R. Macaulay's classic, **The Smoothing of Time Series**, published by the National Bureau of Economic Research (New York) in 1931. This book is now out of print but you can occasionally pick up a copy ($5 to $10) in second-hand book stores and of course you can always consult it at any good library.

The Effect of Moving Averages Upon Random Fluctuations

It should be obvious that the effect of moving averages upon random fluctuations is to average out the irregularities. It should be equally obvious that the more items that are combined into the moving average, the smoother will be your result and the closer it will approximate the average value of the successive numbers. One example should be enough to make this perfectly clear.

In Col. A of Table 7 are shown 30 digits taken at random from the New York City telephone book. For demonstration, these digits have been smoothed by a 3-item moving average, a 7-item moving average, and a centered 10-item moving average. See Fig. 1 on page 307.

It is obvious by inspection that as we increase the number of items of the moving average, the closer all terms of the moving average approach the average value of all the digits, which is 5.07.

TABLE 7.
VARIOUS MOV. AVERAGES OF A SERIES OF RANDOM NUMBERS

	A	B	C	D
ITEM	RANDOM NUMBERS (TAKEN FROM THE TELEPHONE BOOK)	3-ITEM MOV.AVER. OF COL.A	7-ITEM MOV.AVER. OF COL.A	CENTERED 16-ITEM MOV.AVER. OF COL.A
1	6	•	•	•
2	6	6.67	•	•
3	8	6.67	•	•
4	6	4.67	5.00	•
5	0	2.33	5.28	•
6	1	3.00	5.28	•
7	8	5.67	4.43	•
8	8	7.33	3.71	•
9	6	5.33	5.00	5.19
10	2	3.00	5.57	4.81
11	1	4.00	5.14	4.66
12	9	5.00	5.00	4.66
13	5	6.33	5.28	4.84
14	5	5.67	5.00	5.09
15	7	6.67	4.86	5.09
16	8	5.00	4.86	5.06
17	0	2.67	4.86	5.12
18	0	3.00	5.14	5.12
19	9	4.67	4.43	5.03
20	5	7.00	4.28	5.00
21	7	4.67	5.43	5.06
22	2	5.33	6.57	5.09
23	7	5.67	5.28	•
24	8	7.67	4.57	•
25	8	5.33	4.86	•
26	0	2.67	5.57	•
27	0	3.00	5.14	•
28	9	5.33	•	•
29	7	6.67	•	•
30	4	•	•	•

(SEE FIG. 1 ON PAGE 307)

Of course the process of smoothing also has the effect of minimizing cyclic fluctuations that may be present in the data as well as of smoothing out random fluctuations. It would not seem necessary to illustrate this fact at this point.

Weighted Moving Averages

In connection with smoothing a time series, one often gets better (i.e. smoother) results by the use of severak successive smoothings. For example, if one took a 2-year moving average of a 6-year moving average of a 9-year moving average of a time series, one would obtain a much smoother curve than could be obtained by any of these moving averages taken separately.

The compound effect of such a series of consecutive moving averages could be expressed by the following formula:

$$MA_h = \frac{a+3b+5c+7d+9e+11f+12g+12h+12i+ \\ +11j+9k+7l+5m+3n+o}{108}$$

Such a moving average is called a weighted moving average because for each item of the moving average each of the terms is used a different number of times and therefore with different weights.

In the formula described, for any one term of the moving average each of the items g, h, and i have 12 times the effect or weight in the composite as do items a or o, which are used but once.

Macaulay reports upon many formulae which have been developed by various investigators in order to achieve particular purposes. For example, "....take a 3-months moving total of a 5-months moving total of an 8-months moving total of a 12-months moving total of the data. To the results apply the following extremely simple set of weights: + 2, -3, 0, 0, 0, 0, 0, + 3, 0, 0, 0, 0, 0, -3, + 2. Divide the final results by 1440."

A weighted moving average of the sort described above, **with negative weights** near the ends, if properly designed, will overcome the tendency of the ordinary moving average to stay too low at cycle tops and too high at cycle bottoms.

Some of these formulae become rather complicated. For example:

> "Take successively a 3-months moving total of the data, a 5-months moving total, another 5, an 8, and a 12-months moving total. To the results apply the following set of weights: + 1,331,771, - 1,949,056, 0, 0, 0, 0, 0, + 2,175,370, 0, 0, 0, 0, 0, 0, - 1,949,056, + 1,331,771. Divide each of the final results by 6,773,760,000."

In view of the fact that Macaulay has covered the subject so admirably and the further fact that we here are not interested in smoothing as such but only in the smoothing that may result as we use the moving average in the detection and isolation of cycles, it seems unnecessary to give further attention at this time to this aspect of the subject.

B. The Use of Moving Averages in Trend Determination

The moving average is often used to give an approximate measure of the trend of a time series.

Trend may be defined as the tendency of data in a series to increase or decrease over a long period of time. How long is "long" depends upon circumstances.

The table below shows, by means of controlled data, three trend lines and, in connection with each, its 5-year moving average.

The trend shown in Col. A increases by a **constant amount**. The trend shown in Col. C increases by **amounts that get progressively greater** as time goes on. The trend shown in Col. E increases by **amounts that get progressively less** as time goes on. Cols. B, D, and F show a 5-year moving average for each of these three trend lines. The trends, with their moving averages superimposed by means of broken lines, are shown in Fig. 2 on page 307.

TABLE 8.
5-YEAR MOVING AVERAGES OF CONTROLLED DATA SHOWING EFFECT UPON THREE DIFFERENT TYPES OF TREND

	A	B	C	D	E	F
YEAR	A TREND WHICH INCREASES BY A CONSTANT AMOUNT	5-YEAR MOVING AVERAGE OF COL. A	A TREND THAT INCREASES BY AMOUNTS THAT GET GREATER	5-YEAR MOVING AVERAGE OF COL. C	A TREND THAT INCREASES BY AMOUNTS THAT GET SMALLER	5-YEAR MOVING AVERAGE OF COL. E
1ST	40	-	0	-	0	-
2ND	80	-	5	-	80	-
3RD	120	120	15	20	155	150
4TH	160	160	30	35	225	220
5TH	200	200	50	55	290	285
6TH	240	240	75	80	350	345
7TH	280	280	105	110	405	400
8TH	320	320	140	145	455	450
9TH	360	360	180	185	500	495
10TH	400	400	225	230	540	535
11TH	440	440	275	280	575	570
12TH	480	480	330	335	605	600
13TH	520	520	390	395	630	625
14TH	560	560	455	460	650	645
15TH	600	600	525	530	665	660
16TH	640	-	600	-	675	-
17TH	680	-	680	-	680	-

(SEE FIG. 2 ON PAGE 307)

It will be noted by comparing the moving averages with the trend that where, as in Col. A, the trend increases by a constant amount the moving average coincides with it. Where, as in Col. C, the trend increases by amounts that get greater as we go from year to year, the moving average lies above the trend. Where, as in Col. E, the trend increases by amounts that get less from year to year the moving average lies below the trend.

The Geometric Moving Average and Its Use

For growth curves that increase by an increasing amount, such as the curve set forth in Col. C above, we can usually get a better fit by computing the **geometric moving average**.

In fact when the growth increases by increasing amounts such that the **rate** of growth is constant, the geometric moving average will give a perfect fit.

The geometric moving average is merely the **nth root** of the terms **multiplied** together instead of the **nth** of the terms **added** together. For example, for a 5-year **geometric** moving average, instead of successively **adding** together each five consecutive terms and **dividing** by five, you successively **multiply** together each five consecutive terms and take the fifth **root** of the product. The formulae for a 5-year arithmetic moving average and a 5-year geometric moving average are as follows:

The arithmetic moving average:

$$MA_c = \frac{a + b + c + d + e}{5}$$

The geometric moving average:

$$GMA_c = \sqrt[5]{a \times b \times c \times d \times e}$$

Of course, in practice, to get a geometric moving average, one merely looks up the logarithms of the data in a table of logarithms, records them as in Col. B in Table 9, which follows below, computes the arithmetic moving average of the logs, and reconverts by looking up the antilogs, all as demonstrated in the table.

TABLE 9.
COMPUTATION OF A 5-YEAR GEOMETRIC MOVING AVERAGE
CONTROLLED DATA

YEAR	A DATA (TREND, WITH CONSTANT 6% RATE OF GROWTH)	B LOGS OF COL. A	C 5-YEAR ARITHMETIC MOV. AVER. OF THE LOGS	D ANTILOGS OF COL. C I.E. 5-YEAR GEOMETRIC MOV. AVER. OF COL. A
1ST	100.00	2.0000	-	-
2ND	106.00	2.0253	-	-
3RD	112.36	2.0506	2.0506	112.36
4TH	119.10	2.0759	2.0759	119.10
5TH	126.25	2.1012	2.1012	126.25
6TH	133.82	2.1265	2.1265	133.82
7TH	141.85	2.1518	2.1518	141.85
8TH	150.36	2.1771	2.1771	150.36
9TH	159.38	2.2024	2.2024	159.38
10TH	168.95	2.2227	2.2227	168.95
11TH	179.08	2.2530	2.2530	179.08
12TH	189.83	2.2784	2.2784	189.83
13TH	201.22	2.3037	2.3037	201.22
14TH	213.29	2.3290	2.3290	213.29
15TH	226.01	2.3543	-	-
16TH	239.66	2.3796	-	-

As the 5-year geometric moving average is seen by inspection to be the same as the data, there seems to be no need to chart the result.

When the **rate** at which the curve increases is **decreasing**, the geometric moving average will lie below the curve. When the **rate** at which the curve is **increasing**, the geometric moving average lies above the curve. When the **rate** of growth is constant, as in the example above, the geometric moving average lies on the curve. I find the geometric moving average very useful, and use it a great deal.

Even though very few curves grow at an absolutely constant rate of growth, it is true that many growth curves tend to increase this way and are concave upward when plotted on arithmetic paper; in other words, they grow from year to year in an absolute amount which increases with each successive term. This is one reason why, in most cycle analyses it is usually desirable to deal with the logarithms of the data instead of with the data themselves.

C. The Use of the Moving Average in Cycle Analysis

How can a knowledge of moving averages be used to assist you in cycle analysis—that is, (i) to help you detect and separate cycles that may be present in the data you are studying, and (ii) to help you to obtain a more exact knowledge of their characteristics than would otherwise be possible?

Every time you compute a moving average of a time series you affect cycles of every length that may be present in that series. But, you influence cycles of different length in very different ways. And this fact in turn has an effect upon the comparison that you may make between two different moving averages or between the original data and the moving average.

Therefore, where there are several cycles present concurrently in a time series, by a suitable selection of moving averages, you can minimize or even eliminate some of these cycles and leave others virtually unchanged or, if you wish, magnified.

To see how to make these manipulations, you must first examine the effect of moving averages of different lengths upon a perfectly regular cycle that we can use for purposes of demonstration.

This brings up the question of the shape of the cycle that we should use. However, before we begin to talk about cycles and wave shapes, we will need to have in mind a few more definitions of terms. With these out of the way we can return to a discussion of the proper shape of wave to use for our demonstration, without the need of interrupting the discussion to define terms as we go along. From that point we can go on to a discussion of the effect of moving averages upon the wave shape we have chosen.

Definitions of Certain Terms Used in Cycle Analysis

Cycle, coming from a Greek word meaning circle, implies coming around to the place of beginning. Strictly speaking, in the word itself there is no necessary implication of regularity, but the word is often used loosely to denote rhythm or periodicity.

Rhythm, coming from a Greek word meaning measured time, implies a beat, or a tendency toward perfect regularity or periodicity. It

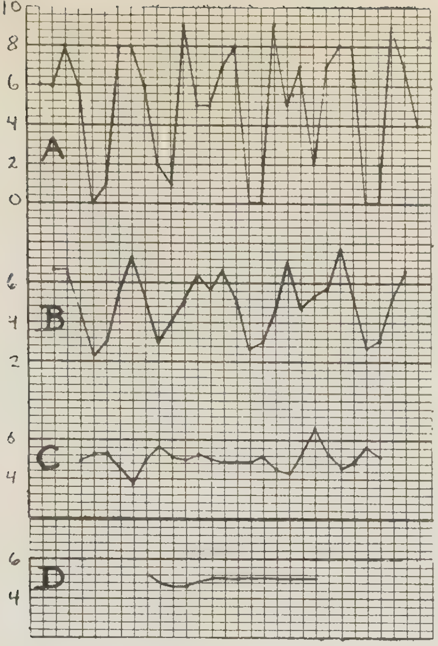


Fig. 1

A. Random Numbers
B. Their 3-Year Moving Average
C. Their 7-Year Moving Average
D. Their 16-Year Moving Average

Note that the longer the moving average, the smoother the curve.

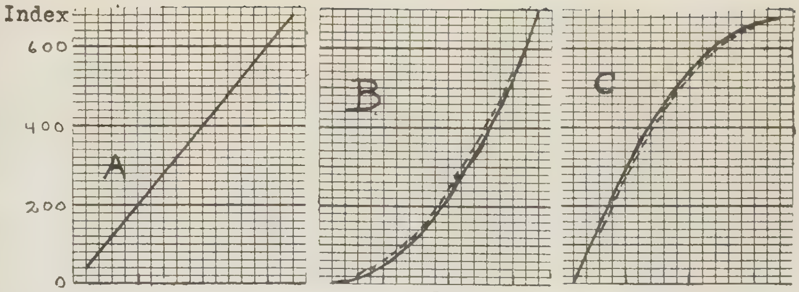


Fig. 2

A. Trend that increases by a constant amount together with its 5-year moving average. (The moving average does not show because it coincides with the trend.)
B. Trend that increases by increasing amounts and, broken line, its 5-year moving average. Note that the moving average lies above the trend.
C. Trend that increases by decreasing amounts and, broken line, its 5-year moving average. Note that the moving average lies below the trend.

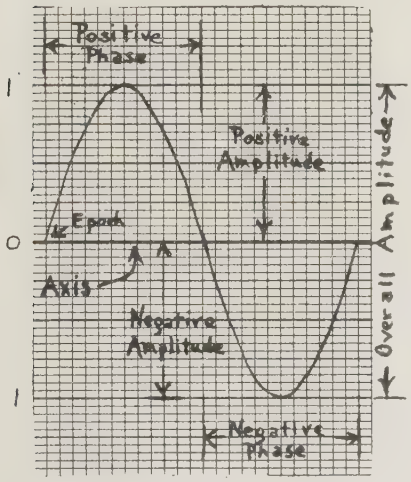


Fig. 3

Sine Wave

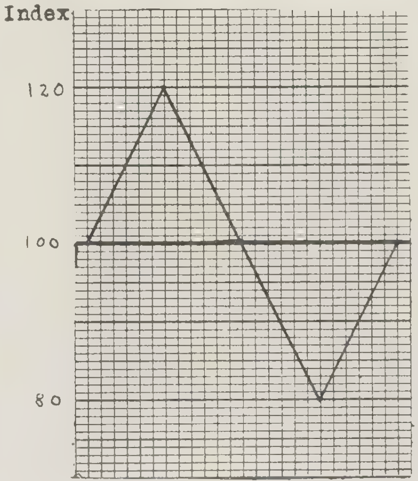


Fig. 4

Rectilinear Wave

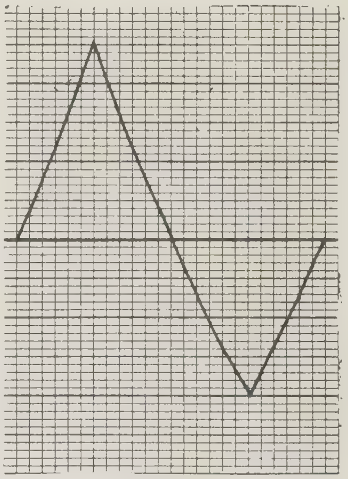


Fig. 5

"Compound Interest" Wave

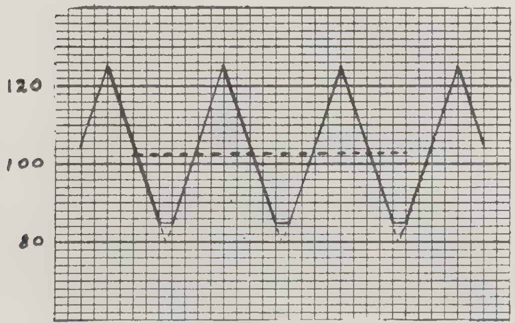


Fig. 6

9-Year Rectilinear Wave
and, broken line,
Its 9-Year Moving Average

Note that a 9-year wave is completely eliminated by a 9-year moving average.

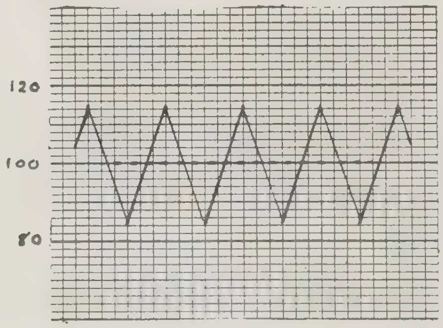


Fig. 7

6-Year Rectilinear Wave
and, broken line,
Its 6-Year Moving Average

Note that a 6-year wave is completely eliminated by a 6-year moving average.

is what we really mean on most of the occasions when we use the word cycle.

Cycle analysis, as we are using the term in this bulletin, should really be called rhythm analysis, as we are concerned with **rhythmic cycles**—cycles that recur with a beat.

Periodicity, in the strict sense, is the quality of being **regularly** recurrent. It is a quality not often found in nature. The ideal cycles that we shall presently construct for purposes of demonstration, however, are true periodicities.

A **wave** is one single cycle or undulation. Waves have **frequency**, **amplitude**, **period**, and, at least when they represent harmonic curves, **phase**.

Frequency is the number of complete vibrations to and fro—i.e. waves—per second. It is a term not used by cycle analysts when dealing with cycles that are over a second in length.

Amplitude is the range on one side or the other from the axis around which the wave oscillates. **Positive amplitude** is the distance above the axis, **negative amplitude** is the distance below the axis, **overall amplitude** is the sum of the positive and negative amplitudes. Amplitude may be expressed in absolute units or as a percentage of the axis or trend.

Period is the interval of time required for a periodic motion to complete a cycle and begin to repeat itself. It is the length of the wave from crest to crest or trough to trough or from some other point on the curve taken as the **epoch**. (The **epoch** is the point on the curve chosen as the beginning of the wave. In physics and astronomy it is usually taken as the point where the curve crosses the axis on its upward motion, but it may be any other point as well.)

Phase, in a simple harmonic curve, is the point or stage in the period to which the oscillation has advanced considered in relation to a standard position or assumed instant of starting. It is measured along the axis, usually in degrees. By extension of meaning, **positive phase** is therefore the part of the wave above the axis or trend, and **negative phase** is the part of the wave below the axis or trend. When the crests (or troughs) of two or more different series of waves come at the same time, the waves are said to be **in phase** with each other. When the crests of one series of waves coincides with the troughs of another series, the series are spoken of as **in reverse phase**.

A **simple harmonic curve** referred to once or twice above, is the curve you would get by tracing the motion of a pendulum upon a piece of smoked paper that was moving at uniform speed at right angles to the direction in which the pendulum was swaying back and forth. It is perfectly simple, regular, and symmetrical and in mathematical study, is usually referred to as a sine curve. A single oscillation is called a **sine wave**. This curve, and many of these definitions are illustrated in Fig. 3 on page 307.

A **rectilinear or saw-tooth wave**, on the other hand, is a wave the sides of which are straight lines; in other words, zigzag. See Fig. 4 on page 307. In electrical engineering a rectilinear wave usually refers to a square wave of this shape ⊓⊔⊓⊔, but the term has a more general application also.

Wave Shapes Usually Found

Because sine waves are so simple in shape, so easy to combine with each other and so satisfactory to handle mathematically, and because they are the shape taken by sound waves and many other kinds of waves with which the physicist deals, it is assumed by many students of cycles in climatology, biology, economics, and other fields that the waves with which they deal **ought** to be sine shape too.

Unfortunately things are not always what they "ought" to be. It has been my experience that waves in economic and biologic time series seem **never** to be sine shape (but this does not mean that the next wave I study might not be of this shape).

It is hard to be sure of the exact shape of a wave. There are almost always variations of length and of amplitude, as we go from one wave to the next. Also there are usually several rhythms present concurrently, and they mix each other up. Finally, there are random factors that enter into the picture which sometimes cannot be removed easily without distorting the wave shape. Therefore I cannot say I am sure of the **exact** mathematical average shape of the waves in any rhythm I have ever studied.

However, if I were forced to express my best guess, I would say that the waves we find in weather, biology, medicine, economics, hydrology, geology, etc., are likely, on the average, to be approximately rectilinear, that is saw-tooth or zigzag. It is not an accident that the " ideal " waves that I have diagrammed in many of the charts that have been published are of this shape.

More exactly, I would put it that the **logarithms** of the data seem, on the average, to conform to a zigzag shape. The result of this fact, of course, is that the average wave shape is saw-tooth when the raw data are plotted on semi-logarithmic paper. This is another way of saying that the sides of the average wave seem to follow the shape of the compound interest curve. That is, the **percentage** rise from the trough to the axis is the same as the **percentage** rise from the axis to the crest

For example, if the trough is at 50 and the axis is at 100, the crest would be at 200 (not 150); one hundred is twice 50, and two hundred is twice one hundred. A wave that follows this law is illustrated in Fig. 5 on page 307.

(The characteristics just described offers another reason why it is usually so highly desirable, in subjecting a series of figures to a rhythm analysis, to convert the raw figures into logarithms before starting work, and to work with them throughout the course of the analysis.)

May I hasten to say that these beliefs are entirely the result of observations as to how the waves in general actually do behave, and are in no sense the result of theories as to how the waves " ought " to behave. I do not yet know enough to talk " oughts. "

A second characteristic of the average waves of the rhythms I have studied is that with most of them the upward movements and the downward movements seem to be symmetrical. That is, the lows tend to fall midway between the highs, and vice versa. This characteristic is so generally true that I have come to suspect as possibly spurious any average wave which, without a reason, fails to conform to this pattern.

On the other hand, I have come across undoubted rhythms where the average waves were very definitely neither symmetrical nor of simple zigzag or compound interest form. It is not safe to try to generalize too rigidly.

In discussing the effect of moving averages upon periodic waves, I have chosen for illustration a perfectly symmetrical rectilinear or zigzag wave, because for small amplitude waves this is a close approximation of the typical form and is in fact seemingly the exact form when the data are converted to logarithms.

The Effect of Moving Averages upon Periodic Waves

1. Simple Waves

a. When the Lenght of the Moving Average is the Same as the Length of the Wave

Suppose yuo have a time series that evidences a perfectly regular 9-year cycle that repeats itself time after time as in Fig. 6 on page 307. The figures for the annual value of such a time series are given below.

Let us compute the 9-year moving average of this series of figures as in Col. C of Table 10.

It is obvious from reference to Col. C that the wave has disappeared and that the moving average is merely a straight line. This straight line has been plotted as a broken line in Fig. 6.

A moment's reflection will explain the reason for this behavior. As the length of the moving average is the same as the length of the wave, the value of the item that is added is always the same as the value of the item that is dropped and, in consequence, the moving average remains unchanged.

It is possible to generalize the above observation and to say that when the moving average has the same length as any perfectly regular wave, its effect is to eliminate the wave completely.

Table 10.
A 9-Year Moving Average of a 9-Year Wave in Controlled Data

	A	B	C
Year	Controlled Data Evidencing a 9-Year Wave	9-Year Moving Tot. of Data	9-Year Mo. Av. of the Data (Col. B ÷ 9; or times 1/9: or times .111111. the reciprocal of 9
1st	105	-	-
2nd	115	-	-
3rd	125	-	-
4th	115	-	-
5th	105	925	102.8
6th	95	925	102.8
7th	85	925	102.8
8th	85	925	102.8
9th	95	925	102.8
10th	105	925	102.8
11th	115	925	102.8
12th	125	925	102.8
13th	115	925	102.8
14th	105	925	102.8
15th	95	-	-
16th	85	-	-
17th	85	-	-
18th	95	-	-

To make the procedure doubly plain and to pave the way for a discussion of a method of separating compound cycles, let us work another example. Table 11, next following, gives a series of figures evidencing a perfectly regular 6-year wave. The data are given, together with their centered 6-year moving average.

Here again you get a complete elimination of the wave. The 6-year moving average of the 6-year wave is merely a straight line. It is charted in Fig. 7 by means of a broken line superimposed upon the 6-year wave with which we started. (See p. 307.)

It should also be obvious that if you had added the 6-year wave to a trend line that increased by constant amounts, the 6-year moving average of the combined wave and trend line would have reproduced the trend free and clear of the wave. (If the trend had increased by increasing amounts, the moving average would have lain above it; if by decreasing amounts the moving average would have lain below it; all as illustrated in an earlier section.)

TABLE 11.
A 6-YEAR MOVING AVERAGE OF A 6-YEAR WAVE

	A	B	C	D
YEAR	DATA EVIDENCING A 6-YEAR WAVE	6-YEAR MOVING TOT. OF THE DATA POSTED TO THE 3RD POSITION	2-YEAR MOVING TOT. OF COL. B POSTED TO THE 2ND POSITION	6-YEAR MOVING AV. OF THE DATA CENTERED (COL. C÷12)
1ST	105	-	-	-
2ND	115	-	-	-
3RD	105	600	-	-
4TH	95	600	1200	100
5TH	85	600	1200	100
6TH	95	600	1200	100
7TH	105	600	1200	100
8TH	115	600	1200	100
9TH	105	600	1,200	100
10TH	95	-	-	-
11TH	85	-	-	-
12TH	95	-	-	-

Waves of Odd and Peculiar Shape

You may wonder if we would get the same result—a straight line—if the wave had some other shape. As long as the repetition is perfectly regular, the shape of the wave makes no difference whatever. This fact is illustrated in the table that follows:

TABLE 12.
A 5-YEAR MOVING AVERAGE OF AN IRREGULAR 5-YEAR WAVE

	A	B
YEAR	DATA EVIDENCING AN IRREGULAR SHAPED 5-YEAR REPETITIVE PATTERN	5-YEAR MOV. AVERAGE OF THE DATA
1ST	100	-
2ND	125	-
3RD	85	100
4TH	105	100
5TH	85	100
6TH	100	100
7TH	125	100
8TH	85	100
9TH	105	-
10TH	85	-

The reason we get a straight line is because the value we add is always the same as the value we drop.

b. When the Length of the Moving Average is An Integral Multiple of the Length of the Wave

A moving average that is two or three (or any other integral multiple) times the length of the wave will also completely eliminate any regular wave.

Thus, if we have a perfectly regular 9-year wave, an 18-year moving average will completely eliminate it, and so will a 27-year moving average, or a 36-year moving average.

If we have a perfectly regular 6-year wave, a 12-year, 18-year, 24-year, or 30-year moving average would give the same result.

You should also note that an 18-year moving average would completely eliminate **both** the 9-year and 6-year waves, because 18 is a multiple of both 9 years and 6 years.

c. When the Moving Average is of a Length That is Different From the Length of the Wave, or from some Integral Multiple of It.

You may wonder what a 3-year moving average of a 9-year wave might look like, or a 5-year moving average, or a 7-year moving average, or an 11-year moving average, or a 13-year moving average.

At this point the shape of the wave begins to make a difference. Let us therefore consider first the effect upon a rectilinear (saw tooth) wave. Such a wave is given in Table 13 on the page following, together with moving averages of various lengths. The various values are plotted in Fig. 8 on page 313.

You will note that as the moving averages get longer they become flatter until, when the length of the moving average equals the length of the wave, the moving average becomes a straight line. When the moving average is longer than the length of the wave, the wave reappears in inverse (upside down) phase. That is, for the 11-year and 13-year moving averages, the 9-year wave reappears with troughs where there were crests in the original data, and with crests in the moving average where we originally had troughs.

The reason for this is very easy to see. The 13-year moving average, for example, centering on a trough, groups together two highs and one low and is therefore obviously **above** the average of one 9-year wave, at time of trough. As we progress in time to a position centering on a crest, the 13-year moving average includes two lows and one high and is therefore obviously **below** the average of one 9-year wave at time of crest.

TABLE 13.
VARIOUS MOVING AVERAGES OF A 9-YEAR WAVE

YEAR	A DATA EVIDENCING A REGULAR RECTILINEAR 9-YEAR WAVE	B 3-YEAR MOVING AVERAGE OF DATA	C 5-YEAR MOVING AVERAGE OF DATA	D 7-YEAR MOVING AVERAGE OF DATA	E 9-YEAR MOVING AVERAGE OF DATA	F 11-YEAR MOVING AVERAGE OF DATA	G 13-YEAR MOVING AVERAGE OF DATA
1ST	105		-	-	-	-	-
2ND	115	115.0	-	-	-	-	-
3RD	125	118.3	113	-	-	-	-
4TH	115	115.0	111	106.4	-	-	-
5TH	105	105.0	105	103.6	102.8	-	-
6TH	95	95.0	97	100.7	102.8	104.1	-
7TH	85	88.3	93	97.8	102.8	105.9	106.5
8TH	85	88.3	93	97.8	102.8	105.9	106.5
9TH	95	95.0	97	100.7	102.8	104.1	105.0
10TH	105	105.0	105	103.6	102.8	102.3	102.7
11TH	115	115.0	111	106.4	102.8	100.5	100.4
12TH	125	118.3	113	107.8	102.8	99.5	99.6
13TH	115	115.0	111	106.4	102.8	100.5	100.4
14TH	105	105.0	105	103.6	102.8	102.3	102.7
15TH	95	95.0	97	100.7	102.8	104.1	105.0
16TH	85	88.3	93	97.8	102.8	105.9	106.5
17TH	85	88.3	93	97.8	102.8	105.9	106.5
18TH	95	95.0	97	100.7	102.8	104.1	105.0
19TH	105	105.0	105	103.6	102.8	102.3	102.7
20TH	115	115.0	111	106.4	102.8	100.5	100.4
21ST	125	118.3	113	107.8	102.8	99.5	99.6
22ND	115	115.0	111	106.4	102.8	100.5	100.4
23RD	105	105.0	105	103.6	102.8	102.3	102.7
24TH	95	95.0	97	100.7	102.8	104.1	105.0
25TH	85	88.3	93	97.8	102.8	105.9	-
26TH	85	88.3	93	97.8	102.8	-	-
27TH	95	95.0	105	100.7	102.8	-	-
28TH	105	105.0	111	-	-	-	-
29TH	115	115.0	-	-	-	-	-
30TH	125	-	-	-	-	-	-

(HIGHS OF EACH CYCLE UNDERLINED)

d. Generalization for Rectilinear Waves

Fig. 9 was worked out by Benjamin Foote and James A. Mitchell of the Hartford Electric Light Company to generalize these facts for rectilinear waves. The chart was drawn for you by Mr. Mitchell. It gives you the percentage of the original amplitude remaining in the moving average for all simple arithmetic moving averages up to four times the length of the wave. (Fig. 9 will be found later in this bulletin on page 314.) This chart is a most useful one for all cycle analysts. I use mine constantly. Let us work out an example or two.

Two Examples

Suppose we have taken a 22-year moving average of a series of figures that contains a 17-year rectilinear (zigzag) wave. How much of the 17-year wave would remain in the 22-year moving average? Twenty-two is approximately 129.4% of 17. Find 129 on the horizontal scale at the bottom of Fig. 9. Construct a perpendicular at this point. This perpendicular will cut the curved line at about minus 16 (read from the scale at position 50. The scale at the extreme left is for values on the horizontal scale from 0 to 50). There will therefore be minus 16% of the 17-year wave remaining in the moving average; that is, the wave in the moving averages will be in reverse phase or upside down from the wave in the original data.

Suppose we had taken a 38-year moving average of the same series of figures. How much of the original 17-year wave would be present in this 38-year moving average? Thirty-eight is 223.5% of 17. Therefore, we find 224 on our horizontal scale, construct a perpendicular. This perpendicular intersects the curve at plus 8 (read from the scale at position 50).. Therefore, we know that 8% of the original amplitude of the 17-year rectilinear wave is still present in the 38-year moving average of these figures. If the amplitude of the moving average should prove to be 4, let us say, we could easily calculate that in the original figures it was 50 because 4 is 8% of 50.

Use of Tables

You may prefer to use a table instead of the chart. If so, you can refer to Table A below.

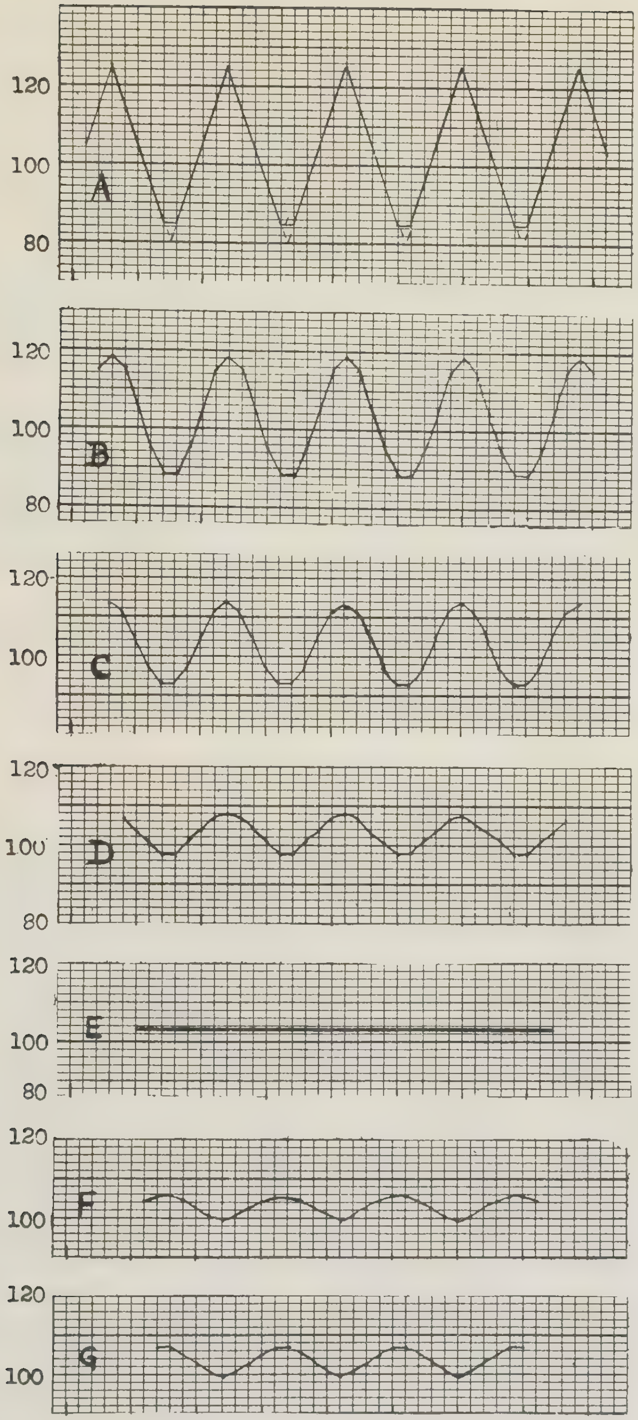
Let us work an example: Suppose we have a 23-year moving average of a regular zigzag shaped 54-year rhythm. How much of the rhythm remains in the moving average? Twenty three divided by 54 is 42.6%. Look up 42.6% in the first column in Table A—the column headed " The length of the moving average expressed as a percentage of the length of the wave. " We find no value for 42.6 but we do find values for 40 and for 45. The percentage of the original amplitude remaining in the moving average for 40 is 60%, for 45 is 55%. By interpolation it is easy to compute that the correct percentage for 42.6% is 57.4%, the required answer.

TABLE A

PERCENTAGE OF AMPLITUDE OF ORIGINAL WAVE REMAINING IN A MOVING AVERAGE,
WHEN THE WAVE IS REGULAR, SYMMETRICAL, AND RECTILINEAR OR SAW-TOOTH IN SHAPE,
FOR VARIOUS LENGTHS OF MOVING AVERAGES UP TO FOUR TIMES THE LENGTH OF THE WAVE.

THE LENGTH OF THE MOVING AVERAGE EXPRESSED AS A PERCENTAGE OF THE LENGTH OF THE WAVE	PERCENTAGE OF ORIGINAL AMPLITUDE REMAINING IN THE MOVING AVERAGE	A CONT'D.	B CONT'D.	A CONT'D.	B CONT'D.	A CONT'D.	B CONT'D.
0	100.	100	.0	200	.0	300	.0
5	95.	105	-4.5	205	2.3	305	-1.6
10	90.	110	-8.2	210	4.3	310	-2.9
15	85.	115	-11.1	215	5.9	315	-4.0
20	80.	120	-13.3	220	7.3	320	-5.0
25	75.	125	-15.0	225	8.3	325	-5.8
30	70.	130	-16.2	230	9.1	330	-6.4
35	65.	135	-16.8	235	9.7	335	-6.8
40	60.	140	-17.1	240	10.0	340	-7.1
45	55.	145	-17.1	245	10.1	345	-7.2
50	50.	150	-16.7	250	10.0	350	-7.1
55	45.	155	-16.0	255	9.7	355	-7.0
60	40.	160	-15.0	260	9.2	360	-6.7
65	35.	165	-13.8	265	8.6	365	-6.2
70	30.	170	-12.4	270	7.8	370	-5.7
75	25.	175	-10.7	275	6.8	375	-5.0
80	20.	180	-8.9	280	5.7	380	-4.2
85	15.	185	-6.9	285	4.5	385	-3.1
90	10.	190	-4.7	290	3.1	390	-2.3
95	5.	195	-2.4	295	1.6	395	-1.2
100	0.	200	0.0	300	0.0	400	0 0

Fig. 8



A. A series of 9-year rectilinear waves

B. Their 3-year moving average

C. Their 5-year moving average

D. Their 7-year moving average

E. Their 9-year moving average

F. Their 11-year moving average

G. Their 13-year moving average

Note how, as the moving average gets longer the waves get flatter until, when the length of the moving average is the same as the length of the wave, they disappear. As the moving average gets longer still, the waves reappear in reverse phase (upside down).

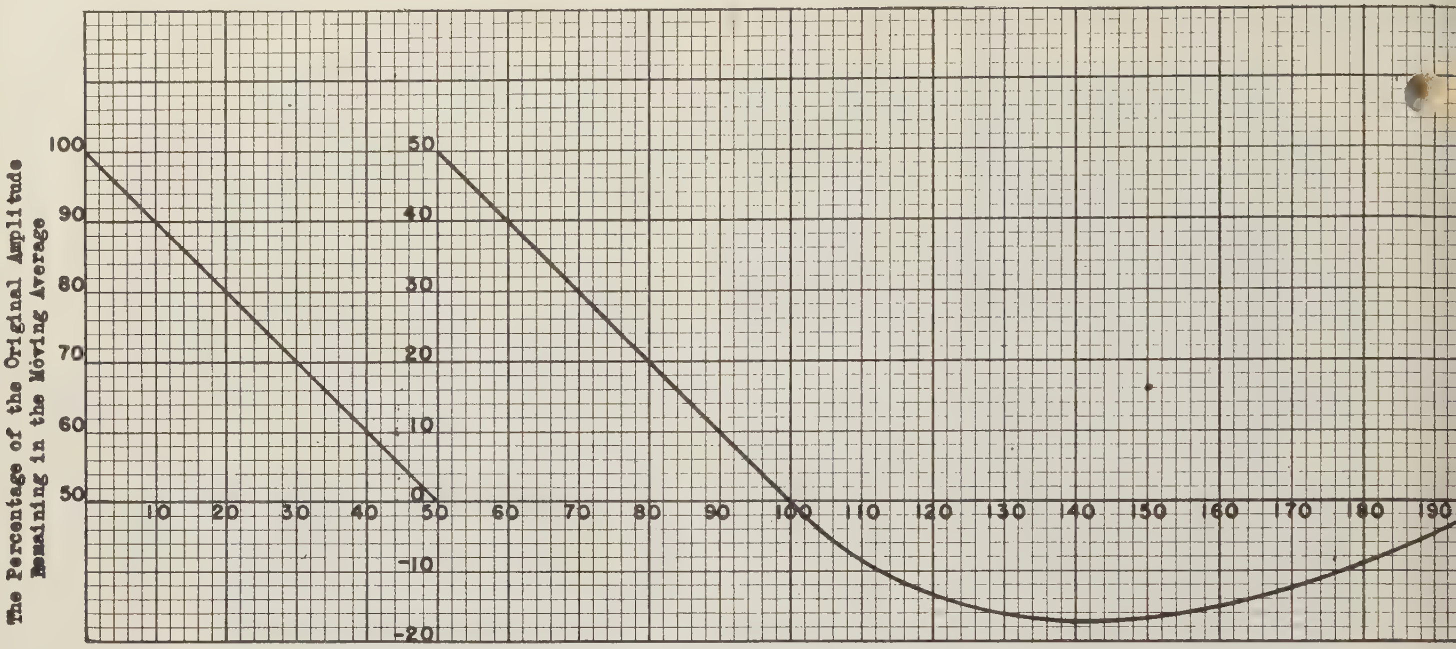


FIG. 9. FOR RECTILINEAR (SAW-TOOTH) WAVES — The Length of th

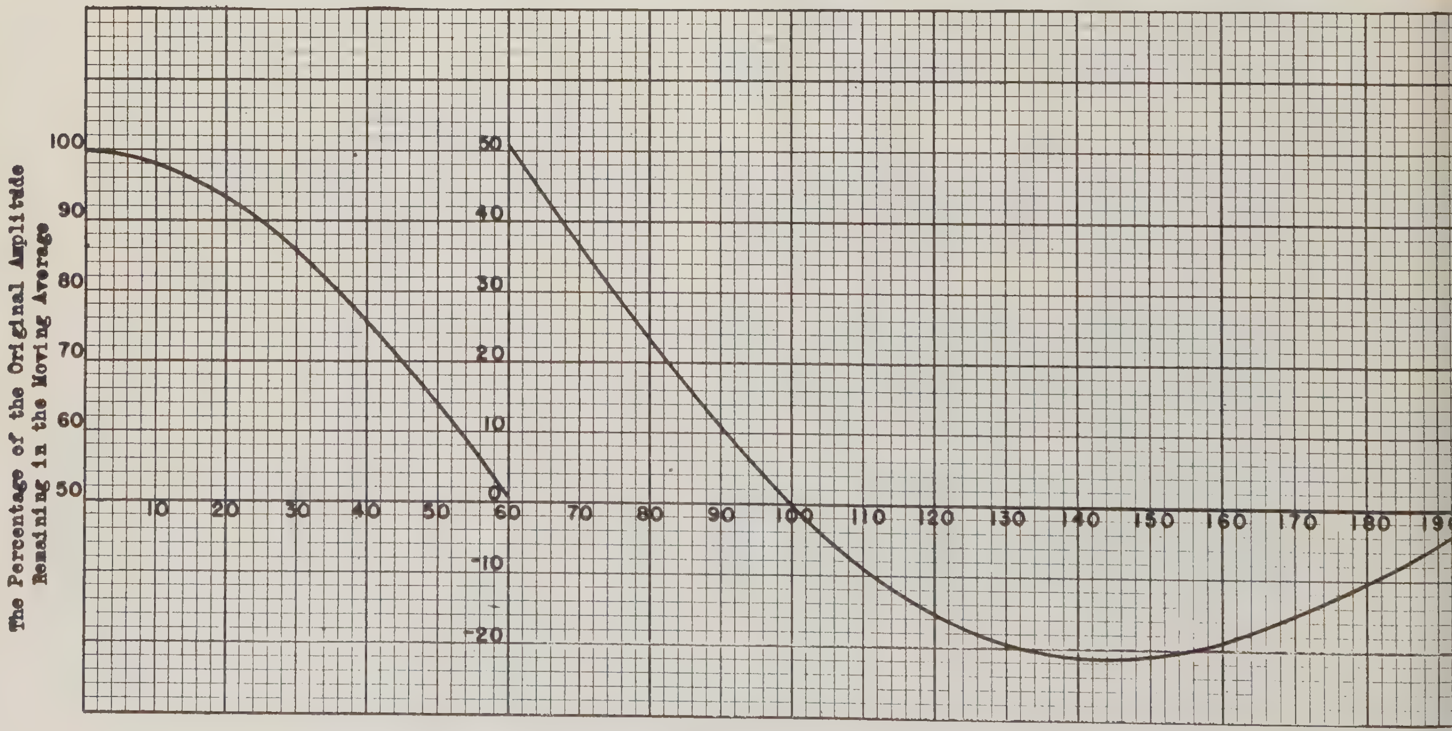
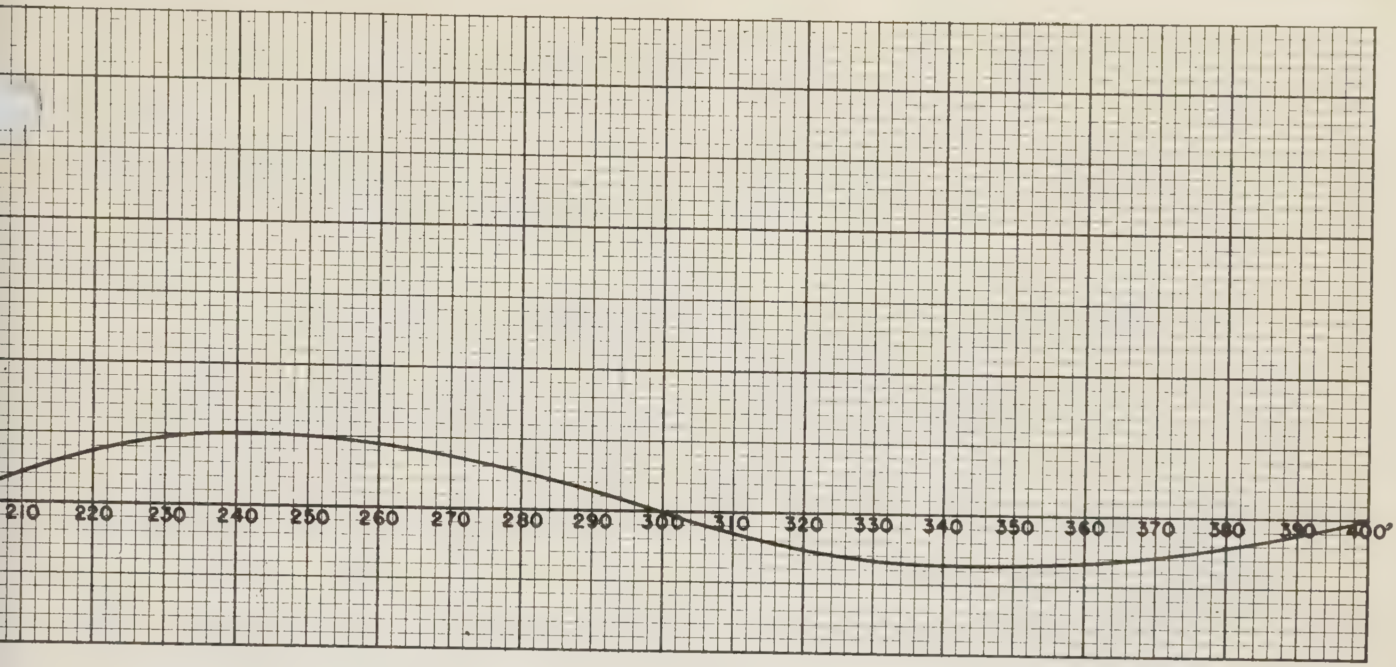
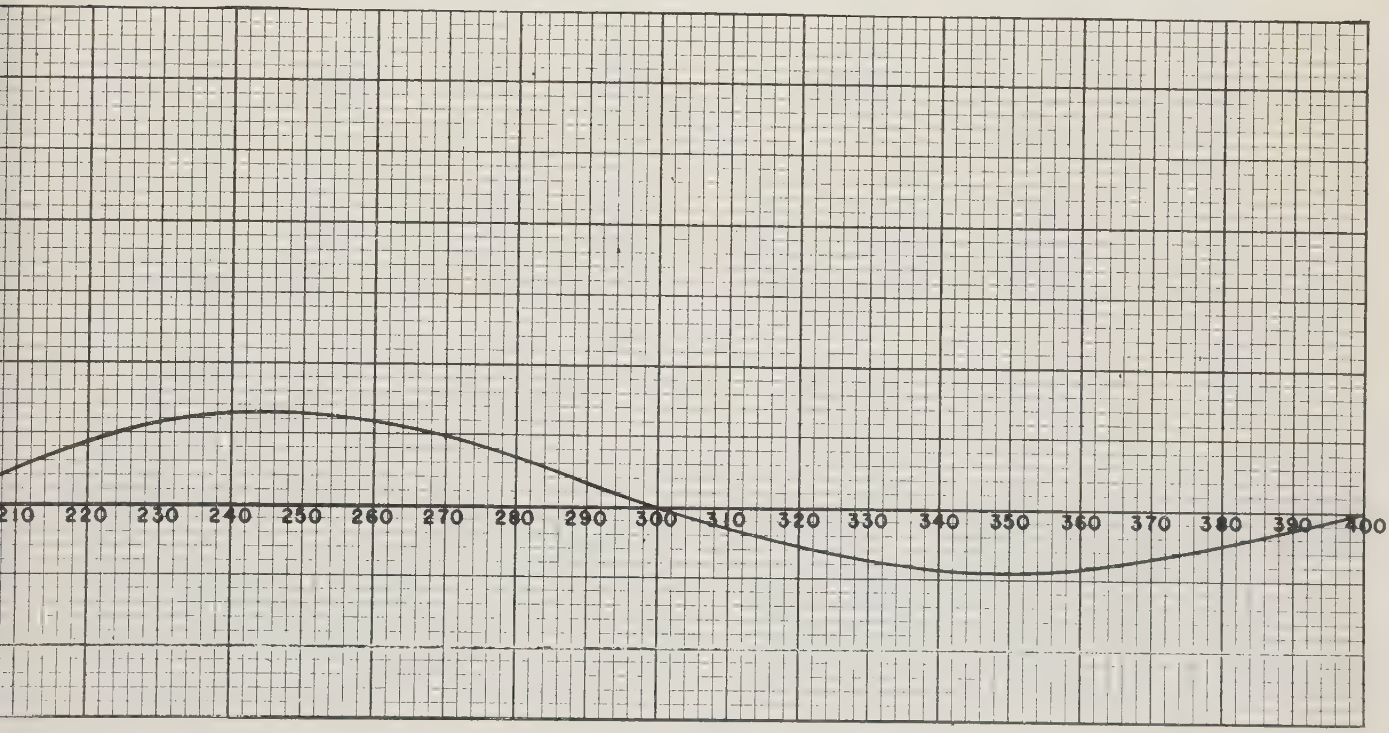


FIG. 10. FOR SINE WAVES — The Length of the



ng Average Expressed as a Percentage of the Length of the Rhythm



e Expressed as a Percentage of the Length of the Rhythm

a. Generalization for Sine Waves

At this point you may ask, does this chart hold true for sine waves and for other waves that are not rectilinear or zigzag shape? No, it does not. Each shape of wave requires a separate diagram. For sine waves Mr. Foote and Mr. Mitchell constructed Fig. 10, inserted previously in this bulletin on page 314, to show the percentage of the original amplitude remaining in the moving average of a sine wave for all given lengths of moving average up to four times the length of the wave.

If you prefer to use a table, refer to Table B below.

TABLE B

PERCENTAGE OF AMPLITUDE OF ORIGINAL WAVE REMAINING IN A MOVING AVERAGE, WHEN THE WAVE IS REGULAR, SYMMETRICAL, AND SINE SHAPED FOR VARIOUS LENGTHS OF MOVING AVERAGES UP TO FOUR TIMES THE LENGTH OF THE WAVE

A THE LENGTH OF THE MOVING AVERAGE EXPRESSED AS A PERCENTAGE OF THE LENGTH OF THE WAVE	B PERCENTAGE OF ORIGINAL AMPLITUDE REMAINING IN THE MOVING AVERAGE	A CONT'D.	B CONT'D.	A CONT'D.	B CONT'D.	A CONT'D.	B CONT'D.
0	.0	100	.0	200	.0	300	.0
5	99.4	105	-4.7	205	2.4	305	-1.6
10	98.4	110	-8.9	210	4.7	310	-3.2
15	96.4	115	-12.6	215	6.7	315	-4.6
20	93.6	120	-15.6	220	8.5	320	-5.8
25	90.1	125	-18.0	225	10.0	325	-6.9
30	85.9	130	-19.8	230	11.2	330	-7.8
35	81.0	135	-21.0	235	12.1	335	-8.5
40	75.7	140	-21.6	240	12.6	340	-8.9
45	69.9	145	-21.7	245	12.8	345	-9.1
50	63.7	150	-21.2	250	12.7	350	-9.1
55	57.2	155	-20.3	255	12.3	355	-8.9
60	50.5	160	-18.9	260	11.6	360	-8.4
65	43.6	165	-17.2	265	10.7	365	-7.8
70	36.8	170	-15.1	270	9.5	370	-7.0
75	30.0	175	-12.9	275	8.2	375	-6.0
80	23.4	180	-10.4	280	6.7	380	-4.9
85	17.0	185	-7.8	285	5.1	385	-3.7
90	10.9	190	-5.2	290	3.4	390	-2.5
95	5.2	195	-2.5	295	1.7	395	-1.2
100	0.	200	0.	300	0.	400	0.

2. Compound Waves

Moving Averages of Time Series Influenced by Two or More Concurrent Cycles

Let us now add together the two series of figures containing the 6-year wave and the 9-year wave respectively that we dealt with above and which were charted in Figs. 6 and 7. This addition is performed in the table on the following page and the result is plotted in Fig. 11 on page 319.

This sum is the kind of a pattern one might expect in the total sales of a company that were equally divided between two products, the sales of one of which fluctuated with a 9-year rhythm, and a second product, the sales of which fluctuated with a 6-year rhythm.

Table 14.
6-Year, 9-Year, and 18-Year Moving Averages of a Compound Wave

	A	B	C	D	E	F
Year	Data Evidencing the 9-Year Cycle	Data Evidencing the 6-Year Cycle	Sum of Col. A and Col. B	6-Year Moving Average of Col. C, Centered	9-Year Moving Average of Col. C	18-Year Moving Average of Col. C, Centered
1st	105	105	210	-	-	-
2nd	115	115	230	-	-	-
3rd	125	105	230	-	-	-
4th	115	95	210	208.3	-	-
5th	105	85	190	204.2	205.6	-
6th	95	95	190	199.2	204.4	-
7th	85	105	190	195.8	201.1	-
8th	85	115	200	195.8	200.0	-
9th	95	105	200	199.2	201.1	-
10th	105	95	200	204.2	204.4	202.8
11th	115	85	200	208.3	205.6	202.8
12th	125	95	220	210.0	204.4	202.8
13th	115	105	220	208.3	201.1	202.8
14th	105	115	220	204.2	200.0	202.8
15th	95	105	200	199.2	201.1	202.8
16th	85	95	180	195.8	204.4	202.8
17th	85	85	170	195.8	205.6	202.8
18th	95	95	190	199.2	204.4	202.8
19th	105	105	210	204.2	201.1	202.8
20th	115	115	230	208.3	200.0	202.8
21st	125	105	230	210.0	201.1	202.8
22nd	115	95	210	208.3	204.4	202.8
23rd	105	85	190	204.2	205.6	202.8
24th	95	95	190	199.2	204.4	202.8
25th	85	105	190	195.8	201.1	202.8
26th	85	115	200	195.8	200.0	202.8
27th	95	105	200	199.2	201.1	202.8
28th	105	95	200	204.2	204.4	-
29th	115	85	200	208.3	205.6	-
30th	125	95	220	210.0	204.4	-
31st	115	105	220	208.3	201.1	-
32nd	105	115	220	204.2	201.1	-
33rd	95	105	200	199.2	-	-
34th	85	95	180	-	-	-
35th	85	85	170	-	-	-
36th	95	95	190	-	-	-

(Highs of each cycle underlined)

Let us now take these figures that evidence this composite wave and compute first a 9-year moving average, second, a 6-year moving average and third, an 18-year moving average as in the table on the preceding page. The various moving averages are plotted in Fig. 11 on page 319. The 9-year moving average has the effect of completely eliminating the 9-year component of the series and shows the 6-year wave in reverse phase, that is to say, with tops where bottoms used to be and vice versa, but with reduced amplitude, all as we would expect from the foregoing discussion.

The 6-year moving average has the effect of completely eliminating the 6-year wave and leaving the 9-year wave in proper phase position (that is, with tops of the moving average where there were tops in the data, and bottoms in the moving average where there were bottoms in the data), but with greatly reduced amplitude.

The 18-year moving average of course eliminates both 6-year and 9-year waves, and would also have eliminated any 4½-year wave (¼ of 18 years), any 3.6-year wave (1/5 of 18 years), any 3-year wave (1/6 of 18 years) and so on if there had been such in the original data. By the same token, it would have revealed any wave longer than 18 years, or shorter waves that were not integral fractions of the length of 18 years, or both, if these had also been present in the data.

Discussion

It should be clear from the foregoing demonstrations that every time you take a moving average of a series of figures, you are performing an operation that has an effect upon the amplitude, and sometimes reverses the phase, of all the regularly recurring waves that may be present in the original figures. It is this fact that prompts the Celticism that one is really not able to start a rhythmic analysis until after one has finished it.

In other words, until one knows the length of all the waves that are present in a series, one is not fully in a position to choose the lengths of the moving averages to use to emphasize some and subordinate others.

Comparison of the Raw Data With with the Moving Average

In the section which began on page 309, you had demonstrated for you the fact that when the length of the moving average is the same as the length of the wave, the effect is the complete elimination of the wave.

Where the original data consist of nothing but a wave (and a horizontal trend line) as in Tables 10 and 11, it is obvious that if we compare the original data with the moving average (which is a horizontal straight line) the result will merely reconstitute the wave in its entirety. This fact is illustrated in Curve EE of Fig. 12 on page 321 and in Col. EE of Table 15 on page 320.

When the moving average is of a length which differs from the length of the wave in the original data, or some multiple of it, some part of the original wave will remain in the moving average. This residue of the original wave remaining in the moving average will be either in phase with the original wave or in reverse phase (upside down). All of this was demonstrated in Table 13 and illustrated in Fig. 8.

When the moving average retains some of the wave **in phase** with the original wave, and when the original data are compared with such a moving average, it should be clear that the difference between the two will show the original wave with **reduced** amplitude. For example, if we have a 9-year wave with an amplitude of 10, and the moving average also contains a 9-year wave coming at the same time with an amplitude of 2, the series of figures evidencing the difference between the two waves will show an amplitude of 8.

When the moving average shows the wave **in reverse phase**, or upside down, and when the original data are compared with it, the difference between the two will show the original wave with **increased** amplitude. For example, if the wave just discussed with an amplitude of 10 were being compared with a moving average that was of such a length that it evidenced a 9-year wave with an amplitude of -1, the wave in the series of figures evidencing the difference would show an amplitude of 11. These facts are all illustrated for the moving averages given in Table 13, in Fig. 12 and in Table 15 on page 320.

It should be noted that the comparisons above have been made by subtraction for the sake of simplicity. In actual practice one ordinarily makes the comparison by division and determines the **percentages** that the original data are of their moving average.

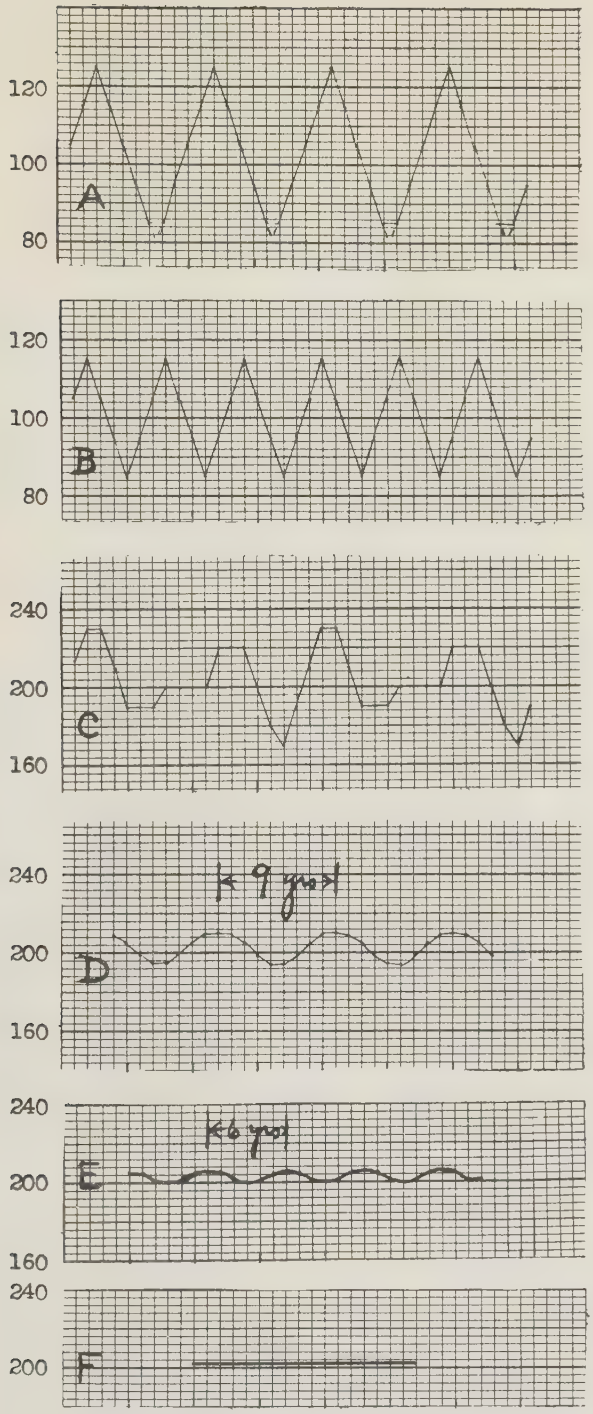
There are several reasons for making the comparisons on a percentage basis. One has already been mentioned—the fact that mostly the waves seem to be the same **percentage** above and below the axis. When one uses percentages on real waves therefore, one tends to get waves that are symmetrical with respect to the axis.

A second reason is that in actual practice, most waves with which one deals are superimposed upon trend lines. That is, the phenomenon with which we deal, let us say the abundance of lynx or the thickness of tree rings or the size of a business, has a long term increase or decrease over a period of time. Experience indicates that the waves that are associated with these various phenomena are usually of approximately constant **percentage** amplitude.

A third reason for making percentage comparisons, even if the trend should be horizontal, is that if there should be other waves it will usually be found that the various waves combine by multiplication and not by addition. They therefore must be unscrambled by division instead of by subtraction.

A fourth reason for making comparison on a percentage basis is that in making projections for most series, one must talk of the waves in terms of percentages. "If the wave continues, the sales of the company at such

Fig. 11



A. A series of 9-year waves

B. A series of 6-year waves

C. Their summation (A + B)

D. A 6-year moving average of the summation (reveals a 9-year wave in phase with the original 9-year wave

E. A 9-year moving average of the summation (reveals a 6-year wave in reverse phase from the original 6-year wave (upside down)).

F. An 18-year moving average of the summation (completely eliminates both waves) and such a time will be 10% above its then trend line." Where the trend line will be at the time must be computed separately.

The only exceptions to the above rule that occurs to me at the moment are (a) the case where the waves in the original data are expressed in plus and minus values, and (b) the case where some of the values of the raw data are zero. In these instances, comparisons between the moving average and the data should usually be made by subtraction.

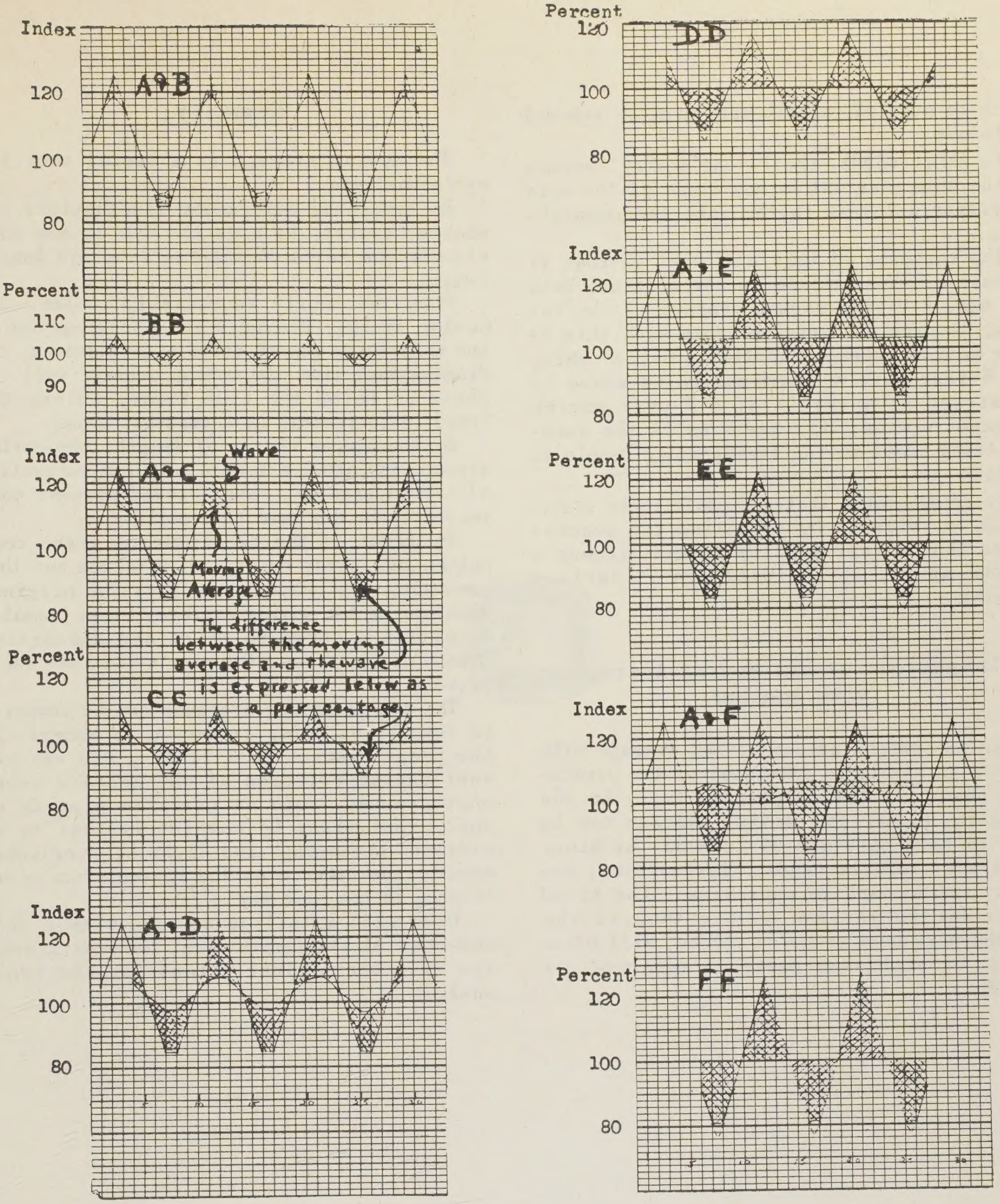
TABLE 15.

COMPARISON OF ORIGINAL CONTROLLED DATA WITH VARIOUS MOVING AVERAGES

YEAR	A DATA EVIDENCING A REGULAR SAW-TOOTH 9-YEAR WAVE	B 3-YEAR MOVING AVERAGE OF THE DATA	BB DATA DIVIDED BY THEIR 3-YEAR MOVING AVERAGE (%)	C 5-YEAR MOVING AVERAGE OF THE DATA	CC DATA DIVIDED BY THEIR 5-YEAR MOVING AVERAGE (%)	D 7-YEAR MOVING AVERAGE OF THE DATA	DD DATA DIVIDED BY THEIR 7-YEAR MOVING AVERAGE (%)	E 9-YEAR MOVING AVERAGE OF THE DATA	EE DATA DIVIDED BY THEIR 9-YEAR MOVING AVERAGE (%)	F 11-YEAR MOVING AVERAGE OF THE DATA	FF DATA DIVIDED BY THEIR 11-YEAR MOVING AVERAGE (%)	G 13-YEAR MOVING AVERAGE OF THE DATA	GG DATA DIVIDED BY THEIR 13-YEAR MOVING AVERAGE (%)
1ST	105	-	-	-	-	-	-	-	-	-	-	-	-
2ND	115	115.0	100.0	-	-	-	-	-	-	-	-	-	-
3RD	125	118.3	105.7	113.0	110.6	-	-	-	-	-	-	-	-
4TH	115	115.0	100.0	111.0	103.6	106.4	108.1	-	-	-	-	-	-
5TH	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	-	-	-	-
6TH	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	-	-
7TH	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
8TH	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
9TH	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
10TH	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
11TH	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
12TH	125	118.3	105.7	113.0	110.6	107.8	116.0	102.8	121.6	99.5	125.6	99.6	125.5
13TH	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
14TH	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
15TH	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
16TH	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
17TH	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
18TH	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
19TH	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
20TH	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
21ST	125	118.3	105.7	113.0	110.6	107.8	116.0	102.8	121.6	99.5	125.6	99.6	125.5
22ND	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
23RD	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
24TH	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
25TH	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
26TH	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
27TH	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	-	-
28TH	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	-	-	-	-
29TH	115	115.0	100.0	111.0	103.6	106.4	108.1	-	-	-	-	-	-
30TH	125	118.3	105.7	113.0	110.6	-	-	-	-	-	-	-	-
31ST	115	115.0	100.0	-	-	-	-	-	-	-	-	-	-
32ND	105	-	-	-	-	-	-	-	-	-	-	-	-

(ALL CRESTS UNDERLINED)

It will be noted both in the chart and in the table that when we compare the original data with their various moving averages that the rhythm that was present in the original data continues present in the comparison. That is, if the wave in the original data is 9 years long, and we run a 7-year moving average through the series (Col. D above) and compare the original data with the 7-year moving average (Col. DD), we get the 9-year rhythm with



The difference between the moving average and the wave is expressed below as a percentage.

Fig. 12

A & B. 9-Year Rectilinear Wave Together With Its 3-Year Moving Average

BB. Data Divided by Their 3-Year Moving Average

A & C. 9-Year Rectilinear Wave Together With Its 5-Year Moving Average

CC. Data Divided by Their 5-Year Moving Average

A & D. 9-Year Rectilinear Wave Together With Its 7-Year Moving Average

DD. Data Divided by Their 7-Year Moving Average

A & E. 9-Year Rectilinear Wave Together With Its 9-Year Moving Average

EE. Data Divided by Their 9-Year Moving Average

A & F. 9-Year Rectilinear Wave Together With Its 11-Year Moving Average

FF. Data Divided by Their 11-Year Moving Average

Note: Note that as the number of terms in the moving average increase and the moving average gets flatter, the original wave reappears more and more in the percentages. When the moving average equals the length of the wave, the wave reappears fully. As the number of items in the moving average increase further, the amplitude of the original wave is magnified.

Note also that the percentages that the data are of their moving averages always evidence waves of the length of the wave in the original data and not the length of the moving average.

which we started, albeit with greatly reduced amplitude.

In other words, the 7-year moving average of the data does **not** in any sense of the word **introduce** a 7-year rhythm into the comparisons.

The converse of this statement is that if we have a 9-year rhythm in the original data and take a 9-year moving average of the series, compare the original data with this 9-year moving average and find a 9-year wave, the 9-year wave we find can in no sense be construed as a **result** of a 9-year moving average, either. This seems to be the hardest thing about moving averages for people to realize.

It is suggested that you prove these statements to yourself by computing the percentages that actual figures which evidence a rhythm are of moving averages of various lengths.

Comparison of One Moving Average with Another

The comparison of one moving average with another is merely an extension of the principles that have been explained fully in the foregoing pages. One moving average can be used to eliminate one or more of the minor waves and minimize random fluctuations, another can be used to approximate the trend line. The comparison of the two, if the lengths have been properly chosen, will often reveal or emphasize waves of intermediate length.

Summary

The moving average is a useful tool for cycle analysts.

By smoothing out random fluctuations and shorter cycles, it aids the eye to see more clearly the waves of intermediate and longer length.

When the length is suitably chosen, the moving average provides an approximation of the underlying growth trend with, however, the disadvantage that the moving average will lie above or below the true trend, unless the trend is increasing by a constant amount.

By the proper choice of length, the moving average can often effect a complete separation of two interacting wave systems present concurrently in the same time series.

By means of the technique of first computing the moving average of a series and then computing the percentages that the original data are of the moving average, it is possible to obtain a curve in which the distorting effect of trend and of longer cycles are minimized.

The use of moving averages can be compared to the use of color filters on a camera. By the proper choice of a filter, you can reveal characteristics of the article being photographed, such as grain in a piece of wood, that might be completely lost in an ordinary photograph and might be overlooked even by the naked eye. The moving average can be used in the same way.

Used with intelligence, and with a full knowledge of its limitations, the moving average is a very valuable tool for the cycle analyst.

MR. NELSON D. GRISWOLD
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FREE!

You have doubtless heard of the cycle analysis of common stock prices made by Mr. Vedder Hughey of Coral Gables, Florida and issued under the trade name of PREDIX.

Mr. Hughey's publication consists of a large (32 x 22) three color chart together with a 24-page booklet.

What the chart shows

The chart shows quarterly average common stock prices from 1854 through the second quarter of 1951, it plots the trend and diagrams 19 different cycles found by Mr. Hughey in these quarterly averages, and it shows the amplitude and timing of each cycle. The chart projects the trend and each of the discovered cycles through 1972. The chart also shows a combination or synthesis of the trend and all of these 19 cycles. It compares this synthesis with actual stock market behavior 1854-1951 and projects the synthesis through 1972 to show the combined effect if the indicated cycles are real and continue.

The booklet

The booklet tells something of the methods used, gives a test forecast 1928-1952 based on data no later than 1915, and gives tables showing the actual numerical value of the index (in logs), the combination of cycles and trend, and the residue. It also gives dates and numerical values of the turning points of each of the 19 waves.

The job impresses me as honest, scholarly, competent, and worthy of the most careful consideration by all serious students of cycles.

The chart and booklet ordinarily sell for $10.

However, to help the Foundation along Mr. Hughey will send you this $10 chart and booklet absolutely free if you send in to us a gift membership in the Foundation ($10) for some friend along with the renewal of your own membership.

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